Statistical Optimization of Quality Improvement by Using Taguchi Methods With Application

A Dissertation

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To My Parents' spirits,
To Shireen,
Rekar,
Rasan,
and Zene.

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Abstract

Design of a good information system based on several characteristics is an important requirement for successfully carrying out any decision making activity.

In many cases although a significant amount of information is available, we fail to use information in a meaningful way .As we require high quality products in day –to –day life ,it is also necessary to have high quality information systems to make robust decisions or predictions .

Usually, the required information is to be extracted from many variables (characteristics) that define a multidimensional system .

A multidimensional system could be an inspection system or a medical diagnosis system and it is important to have a measurement scale by which the degree of abnormality (severity) can be measured .In case of inspection system the degree of abnormality refers to the level of acceptance of a product and in medical diagnosis , the degree of abnormality refers to the severity of a disease .

This dissertation presents methods to develop a multidimensional measurements scale by integrating mathematical and statistical concepts such as Mahalanobis Distance (MD), Mahalanobis –Taguchi system (MTS) and other methods such as the Mahalanobois- Taguchi-Gram-Schmidt (MTGS) methods .

This dissertation includes the application of statistical methods and some special Taguchi methods.

The application data include medical and chemical data, these data were used in the application chapter which includes the result of chemical analysis of water compound.

The Taguchi method based on statistical design of experiments is applied of the parameter design to establish optimum process settings or design parameters.

The experiment was carried out by using a standard Taguchi's experimental plan with denotation $L_{16}(2^{16})$ orthogonal array to optimize the process parameters by the analysis of Signal to Noise (S/N) ratio .

In this dissertation it is concluded that the value of MD in the two methods MD-MTS and MD-MTGS are equal, and determining the important variables in the analysis for two problems and by using the S/N ratio it is seen that the variables (X10 , X12) are not very important during the analysis.

To identify the effect of several control factors and to minimize of the quality characteristic variation due to uncontrollable parameters(Noise) .

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Terms and Symbols

Symbols	Description
MD	Mahalanobis Distance
MS	Mahalanobis Space
MTS	Mahalanobis Taguchi System
MTGS	Mahalanobis Taguchi –Gram-Schmidt process
µ	Mean
Ó	Standard Deviation for population
SD	Standard Deviation for sample
OA	Orthogonal array
S_e	Error Sum of Squares
S_T	Total Sum of Squares
T	Threshold
V_e	Error Variance
Z_i	Standardized Variables
S/N	Signal-To-Noise Ratio
U_i	Gram-Schmidt Variables
MINITAB	A Statistical Software
A	Loss Corresponding to the Distance A
A_0	Loss associated with functional
C^{-1}	Inverse of Correlation Matrix
$L_n(X^Y)$	Representation of an Orthogonal Array

Chapter One

General Introduction

Chapter One

Section One

General Introduction

1-1 Introduction

Mankind has always had fascination with quality, today's technology bears testimony to man's incessant desire to provide a higher level of quality in product and services to increase market share and profits.

To lead a better life we need various kind of products and services ,which are produced by human work and shared by all people .

To live a better life we need a different kinds of products and services, which results in humanitarian work and shared with all people, and that any production process, irrespective of whoever the target output for the service of a humanitarian or non-humanitarian, peaceful or non-peaceful dependent quality, good in this sense, use the human mind in this direction, the best types and styles in the production process in order to achieve good quality that is acceptable by the customer and has acceptable economy.

To compete successfully in the global market place, organizations must have the ability to produce a variety of high quality, low cost products that fully satisfy customer's needs.

Tomorrow's leaders will be those companies that make fundamental changes in the way organizations develop technologies and design and produce products.

It is known that any production process will not pass without quality control, in our daily lives there are many kinds of new ways to control the quality, the first application of statistical methods of the problem of the newly discovered quality control and Shewhart of Bell Telephone Laboratories, issued a memorandum on May 16,1924 that appeared to draw a chart of modern control. Walter Andrew

Shewhart (pronounced like "Shoe-heart", March 18, 1891-March 11, 1967) was a physicist, engineer and statistician, sometimes known as the father of statistical quality control, he contributed to the construction of new scientific methods.

From that date many statisticians contributed to improve methods and new advance methods, one of these contributions was made by the renowned engineer Dr. Genichi Taguchi.

To produce high quality products, the variability in the processes must first be reduced, variability can be accurately measured and reduced only if we have a suitable measurement system with appropriate measures to make accurate predictions or decisions. Usually the required information is to be extracted from many variables (characteristics) that define a multidimensional system that could be an inspection system, a medical diagnosis system a weather forecasting system, or a university admission system.

As we encounter these multidimensional systems in day-to–day life, it is important to have a measurement scale by which the degree of abnormality (severity)can be measured. In the case of medical diagnosis the degree of abnormality refers to severity of a disease, in an inspection system, the degree of abnormality refers to the level of acceptance of a product.

Several multidimensional multivariate methods are being used in multidimensional applications. In this dissertation we present methods scale by integrating mathematical and Mahalanobism distance (MD), Mahalanobis-Taguchi system (MTS) and Gram-Schmidt orthogonalization method with principles of robust engineering (or Taguchi methods). These methods are developed by visualizing the multidimensional system in a different way.

The measures and produces used in these methods are data analytic and do not depend on the distribution of the characteristics that defining the system.

The Mahalanobis –Taguchi system (MTS) developed by Dr.Taguchi is a new technology which combines Mahalanobis distance, orthogonal arrays (OA's)and Signal to-Noise (SN) ratio which is used to assess the effectiveness of the system.

Taguchi defined new methods and demonstrated how these techniques can be applied and how to build into manufacturing organizations the flexibility that minimizes product development cost reduces product time of the market and increases overall productivity.

Outstanding flexibility reduces the product cost and reduces time product and increases productivity.

Statisticians have welcomed professional goals and improvements that resulted from the Taguchi methods, particularly through the development of Taguchi designs to study change method, or the theory of Dr. Taguchi which has many uses, it can be used in all the fields of industry, production services and medical, uses especially in the advanced industrial countries.

This dissertation includes the application of statistical methods and some special Taguchi methods.

The application data include medical and chemical data, these data were used in the application chapter which includes the result of chemical analysis of water compound, was collected from (Directorate Of Health Sulaimanyah-Directorate Of Prevention Health) are taken in different regions within the province of Sulaymaniyah..

Chapter one section one and section two include the general introduction, literature review, Taguchi methods and loss function.

Chapter one section three is about the theoretical parts MD distance, (MTS, MTGS, OA and S/N ratios).

Chapter Two section one is the application chapter which includes the following :

I- Using the (Mahalanobis–Taguchi Syastem (MTS) and Mahalanobis –Taguchi Gram –Schmidt process (MTGS)) methods .

A comparison is made between these two methods MTS and MTGS .

The application provided in this chapter includes the chemical analyses of Components of water data which conclude two kinds of data (Normal and Abnormal).

II- Analyzing the results of MTGS and then identifying the abnormal data and normal data .

III- Removing the abnormal data then repeating the process after that analyzing the results of MTGS.

Chapter two section two deals with using orthogonal arrays (OA) and Signal-to noise (S/N) ratio for dimensionality reduction is demonstrated. In robust engineering OA is used to estimate the effect of several factors and the effect of interactions by minimizing the number of experiments.

In this chapter the S/N ratio and response effect level are calculated, the result of S/N ratio is analyzed and response effect level of the values is optimized. The abnormal data are analyzed to identify the useful variables.

Chapter Three deals with the conclusions and recommendations.

1-2 What Is Quality? [7][16]

The word 'quality' is often used to signify the 'excellence' of a product or service .In some manufacturing companies quality may be used to indicate that a product conforms to certain physical characteristics set down with a particularly 'tight' specification. But if we are to manage quality, it must be defined in a way which recognizes the true requirements of the 'customer [7] **,**

"Quality: Is an inherent or distinguishing characteristic, a degree or grade of excellence." (American Heritage Dictionary, 1996)

"Quality: The totality of characteristics of an entity that bears on its ability to satisfy stated and implied needs" (ISO 8402).

"Quality: Do the right thing, and do things right all the time." [16]

1-3 What is control? [3][6][7]

It is known that the quality is used in every day life, we must make some obvious points .And it is clear that Product quality not equal product quantity. This is a cultural issue .

All the processes can be monitored and brought 'under control' by gathering and using data. This refers to measurements of the performance of the process and the feedback required for corrective action, where necessary requirements, one can ask the question: ‘Are we doing the job correctly’? and the process controls on in processes – in other words, the operator of the process has been given the ‘tools’ to control it. If one now re-examines the first question: ‘Have we done it correctly?’, one can see that, if people we have been able to answer both of the questions: ‘Can we do it correctly?’(capability) and ‘Are we doing it correctly?’ (control) with a ‘yes’, they must have done the job correctly – any other outcome would be illogical. By asking the questions in the right order, one has removed the need to ask the ‘inspection’ question and replaced the strategy of detection with one of prevention. This concentrates attention on the front end of any process – the inputs – and changes the emphasis to making sure the inputs are capable of meeting the requirements of the process.

This is a managerial responsibility and these ideas apply to every transformation process, which must be subjected to the same scrutiny of the methods, the people, the skills, the equipment and so on to make sure they are correct for the job. The control of quality clearly can take place only at the point of transformation of the inputs into the outputs, the point of operation or production, where the letter is typed or the artifact made. The act of inspection is not quality control. When the answer to ‘Have we done it correctly?’ is given indirectly by answering the questions on capability and control, then we have assured quality and the activity of checking becomes one of quality assurance – making sure that the product or service represents the output from an effective system which ensures capability and control.[6][7]

1-4 What is a process? [3][6][7]

A process is the transformation of a set of inputs, which can include materials, actions, methods and operations, into desired outputs, in the form of products, information, services or “generally” results. In each area or function of an organization there will be many processes taking place. Each process may be analyzed by an examination of the inputs and outputs. This will determine the action necessary to improve quality.

The output from a process is that which is transferred to somewhere or to someone–the customer. Clearly, to produce an output which meets the requirements of the customer, it is necessary to define, monitor and control the inputs to the process, which in turn may have been supplied as output from an earlier process. At every supplier–customer interface there resides a transformation process and every single task throughout an organization must be viewed as a process in this way.

To begin to monitor and analyze any process, it is necessary first of all to identify what the process is, and what the inputs and outputs are. Many processes are easily understood and related to known procedures,

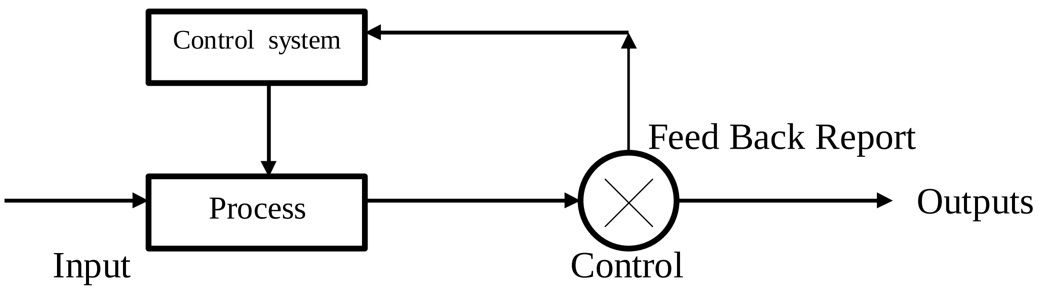


Fig. (1-1) **Shewhart monitor of the control system** [3]

1-5 The historical development of the modern quality method [1][6][7][16]

Corporations routinely apply statistical methods, partly in response to accountability issues, as well as due to the large volume of items produced, miniaturization, and mass customization. An overview of selected statistical methods is provided in Figure (1-2)the origin of these methods dates back at least to the invention of calculus in the 1700s. Least squares regression estimation was one of the first optimization problems addressed in the calculus-optimization literature.

In the early 1900s, statistical methods played a major role in improving agricultural production in the U.K. and the U.S. These developments also led to new methods, including fractional factorials and analysis of variance (ANOVA) developed by Sir Ronald Fisher (Fisher 1925)

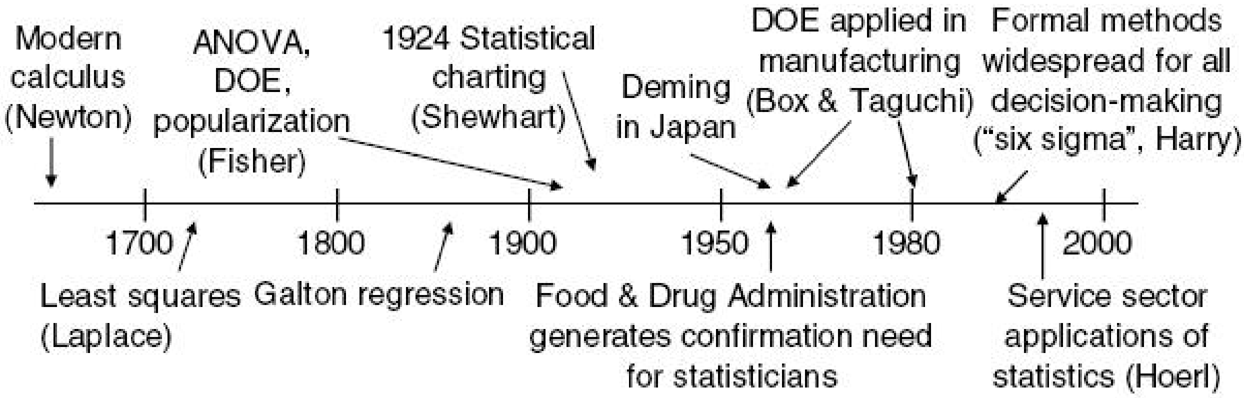


Figure (1-2) Timeline of selected statistical methods [1]

1-5-1- Statistical process control (1924)

Statistical process control (SPC) is the application of statistical techniques to control a process. In 1924, Walter. A. Shewhart of Bell.Telephone Laboratories developed a statistical control chart to control important production variables in the production process. This chart is considered as the beginning of SPC and one of the first quality assurance methods introduced in modern industry. Shewhart is often considered as the father of statistical quality control because he brought together the disciplines of statistics, engineering, and economics.

1-5-2 Design of experiment (late 1930s)

Design of experiment (DOE) is a very important quality tool in current use. DOE is a generic statistical method which guides design and analysis of experiments in order to find the cause-and-effect relationship between "response" (output) and factors (inputs). This relationship is derived from empirical modeling of experimental data.

In the 1930s, Sir Ronald Fisher, a professor at the University of London, was the innovator in the use of statistical methods in experimental design. He developed and first used analysis of variance (ANOVA) as the primary method in the analysis in experimental design.

DOE was first used at the Roth Amsted Agricultural Experimental Station in London. The first industrial applications of DOE were in the British textile industry. After World War II, experimental design methods were introduced in the chemical and process industries in the United States and Western Europe.

1-5-3 Acceptance sampling (1940)

In the production stage, quality assurance of incoming parts from other suppliers is also important, because defective parts could certainly make a defective final product. Obviously, 100% inspection of all incoming parts may identify defective parts, but this is very expensive.

The acceptance sampling plan was developed by Harold F. Dodge and Harry G. Romig in 1940. Four sets of tables were published in 1940: single-sampling lot tolerance tables, double-sampling lot tolerance tables, single-sampling average outgoing quality limit tables, and double-sampling average outgoing quality limit tables.

1-5-4Tools for manufacturing diagnosis and problem solving (1950s)

Statistical process control (SPC) is a process monitoring tool. It can discern whether the process is in a state of normal variation or in a state of abnormal fluctuation. The latter state often indicates that there is a problem in the process. SPC cannot detect what the problem is. Therefore, developing tools for process troubleshooting and problem solving is very important. There are many tools available today for troubleshooting; however, Kaoru Ishikawa's seven basic tools for quality and Dorian Shainin's statistical engineering deserve special attention.

1-6 The kinds of tools:[3][6]

1-6-1Total quality management (TQM) (1960)

TQM is a management approach to long-term success through customer satisfaction and is based on the participation of all members of an organization in improving processes, products, services, and the culture in which they work. Total quality management emphasized the ideas of Shewhart and the role of data in management decision-making. The methods for implementing this approach are found in the teachings of such quality leaders as W. Edwards Deming, Kaoru Ishikawa , Joseph M. Juran, and many others. W. Edwards Deming was a protégé of Dr.Walter Shewhart. Edward Deming is credited with playing a major role in developing so-called “Total Quality Management” (TQM).

1-6-2 Robust engineering/Taguchi method (1960s in Japan, 1980s in the West)

Dr.Genich Taguchi system of quality engineering is one of the most important milestones in the development of quality methods. Taguchi method, together with QFD, extended the quality assurance activities to the earlier stages of the product life cycle. Taguchi’s quality engineering is also called the robust design method.

1-6-3 Quality function deployment (QFD) (1960 in Japan, 1980s in the West))

Quality function deployment (QFD) is an effective quality tool in the early design stage. It is a structured approach to defining customer needs or requirements and translating them into specific plans to produce products to meet those needs. The “voice of the customer” is the term used to describe these stated and unstated customer needs or requirements

1-6-4 Theory of Inventive Problem Solving, TRIZ (1950s in Soviet Union, 1990s in the West)[16][27]

TRIZ (Teoriya Resheniya Izobreatatelskikh Zadatch) theory was created by The Russian inventor –Genrish S. Altshuller and his research fellows in 1946. They have analyzed and integrated 1500000 patents since 50 years. TRIZ is a definition for theory of Inventive Problem Solving (TIPS). TRIZ is a combination of methods, tools, and a way of thinking (Darrel Mann 2002). TRIZ is another tool for design improvement by systematic methods to foster creative design practices.

TRIZ has five key philosophical elements. They are:

1- Ideality.

2- Functionality

3- Resource.

4- Contradictions

5- Evolution.

1-6-5 Axiomatic design (1990)

Axiomatic design is a principle-based method that provides the designer with a structured approach to design tasks. In the axiomatic design approach, the design is modeled as mapping between different domains

1-7 Literature Review

Taguchi Methods are a statistical methods that are widely used in many fields, especially in medical felds, engineering, production, etc.

In 1991 Resit Unal ,and Edwin B.Dean [43] used the Taguchi Approach to present an overview of the Taguchi methods for improving quality and reducing cost, they described the current state of applications and its role in identifying cost sensitive design parameters.

In the same year Kevin N. , Ottoy and Erik K.[32] explained that the Taguchi method of product design is an experimental approximation to minimizing the expected value of target variance for certain classes of problems. Taguchi's method is extended to designs which involve variables each of which has a range of values all of which must be satisfied necessity, and designs which involve variables each of which has a range of values.

Chen and Tam, in1996[20] introduced the Taguchi method of experimental design in optimizing process parameters for micro engraving of iron oxide coated glass using a Q-switched Nd:YAG laser

In 1997 Michael, W. Trosset[36] intended to a facilitate dialogue between engineers and optimization about the efficiency of Taguchi methods for robust design ,especially in the context of design by computer simulation .

In 2000, Taguchi and J.Rajesh [44] introduced new trends in multivariate diagnosis to show that Taguchi methods are useful in multidimensional system application and used the Mahalanobis distance (MD)for constructing a measurement scale for multidimensional system and the principles of Taguchi Methods for optimizing the system it refers to the procedure as the Mahalanobis – Taguchi System (MTS). The measure and methods used in MTS are data analytic rather than usual probability based inference. In this paper detailed procedure of MTS is explained with two case studies.

M.Mohamad, and Z.Ahmad in 2002[38] used Taguchi Robust Design Technique to rank several factors that may affect the deposition rate in order to formulate the optimum electroless nickel bath, and the Taguchi orthogonal array L9 was used for the experimental design with three levels of consideration for each factor, they used Taguchi's signal-to-noise ratio (S/N) and analysis of variance. The results showed that the most notable factor influencing the deposition rate was the PH, followed by the concentration of nickel salt, reducing agent and completing agent. The optimum bath formulation was then predicted based on these results. The key word are Electroless nickel plating.

In 2004 Miran, Hvalec, and Peter. G.[35] used Taguchi method to plan a minimum number of experiments, after identifying the working levels of the design factors and the main performance characteristics of the product under the study of Experimental Design of Crystallization Processes by using Taguchi Method.

Li Qi, Chadia, S.Mikhael and W. Robert in 2004[33] incorporated the Taguchi method into the to study of the sensitivity analysis of a model is of investigating how outputs vary with changes of input parameters, in order to identify the relative importance of parameters and to help in optimization of the model .

In the same year Huei-Chun Wang , Chih-Chou Chiu[47] have developed the Mahalanobis –Taguchi System in data classification of two sets which are analyzed to demonstrate the effectiveness of the MTS. Implementation results reveal that the MTS outperforms traditional discriminate analysis methods in addition to several important issues regarding the MTS.

In 2005 Derong Liu and Ying Cai, [25] presented a new algorithm that applies the Taguchi method to solve the economic dispatch problem with nonsmooth cost functions. Taguchi method involves the use of orthogonal arrays in estimating the gradient of the cost function. The Taguchi method has been widely used in experimental designs for problems with multiple parameters where the optimization of a cost function is required.

In the same year Danaco N. and F.Zehra [23] considered Taguchi Techniques for 2k Fractional Factorial Experiments .Connection between the linear graph method and graph theory is investigated. They used orthogonal arrays, linear graphs, and triangular tables in the design of experiments.

S. S. Madaeni, and S. Koocheki, in 2006,[34] applied Taguchi method in the optimization of wastewater treatment in Exira pharmaceutical .The pilot plant consists of two spiral wound RO elements. The RO train was configured in series. Trial runs were conducted at different operating conditions including pressures, temperature and concentration.

Younes M, and Rahil M. in 2006[51] studied the choice genetic parameters with Taguchi Method Applied in Economic Power Dispatch, The application of genetic algorithm (GA) and the Taguchi Method. The economic power dispatch is a non-linear optimization problem with several constraints.

Chao-Ton Su and Yu-Hsiang H. in 2007[45] in their study An Evaluation of the Robustness of Mahalanobis-Taguchi System state that (MTS) is a diagnostic and forecasting technique for multivariate data. To verify the robustness of MTS for imbalanced data and to compare MTS with several popular classification techniques. .

In 2007 Rashidi M. and Hasbullah I.[37] applied Taguchi Method to analyze the nodularisation effect on the parameter of ductile iron castings. An attempt has been made to obtain optimal settings of the casting parameters, in order to yield the optimum condition for in mould treatment of ductile iron castings.. The result indicates that the selected parameters significantly affect the nodularisation of ductile iron castings.

Wei-Chung Weng and A. Elsherbeni [48] in the same year described a new global electromagnetic optimization technique using Taguchi's method and applied it to linear antenna array design. Taguchi's method was developed on the basis of the orthogonal array (OA) concept, which offers systematic and efficient characteristics.

Elizabeth A. Cudney and Taguchi, in 2007[21] discussed Taguchi System and Neural Network for multivariate pattern recognition to compare the ability of the Mahalanobis- Taguchi System and a neural-network to discriminate using small data sets.

In 2007 L. A. Dobrzañski, J. Domaga, J. F. Silva [26] applied Taguchi method to find the optimum parameters to produce Twintex tubes by filament winding.

Gusri and Nagi, A. in 2008[30] introduce a study to show that Taguchi offers a simple and systematic approach to optimize performance, quality and cost. Taguchi optimization methodology is applied to optimize cutting parameters in turning Ti-6Al-4V ELI with coated and uncoated cemented carbide tools. The turning parameters evaluated are cutting speed, feed rate, depth of cut and type of cutting tool.

In the same year Chen, Y. and Phillips, J. [19], presented a new approach to detect potential ECU application software abnormal behavior based on the Mahalanobis Distance, the Mahalanobis-Taguchi System.

Dr. Osama S. and Hussam L. in 2009[39] used Taguchi Method to optimize the casting of Al–Si /Al2O3 composites. Composites were prepared by vortex technique using three different parameters, stirring time, stirring speed, and volume .Taguchi method was successful in predicting the parameters that give the highest properties and the volume fraction was the most influential parameter on the tensile strength and hardness results of castings .

Uður Eºme in 2009 [28], in the confirmation tests indicated that it is possible to increase tensile shear strength significantly by using the Taguchi method. The experimental results confirmed the validity of the used Taguchi method for enhancing the welding performance and optimizing the welding parameters in the resistance spot welding process.

Tian-Syung Lan in 2010[46]. introduced fuzzy Taguchi deduction optimization on multi-attribute (computer numerical control-cnc) turning. Four parameters with three levels are considered to optimize the multi-attribute finish CNC (computer numerical control) turning based on ($L_9(3^4$) orthogonal array.

M. Demirc, and I. Asiltürk in 2011[24] considered optimization of fatigue life parameters with Taguchi Method. Their study aims to attain best fatigue life parameters effects of parameters such as filament angle. Surface crack depth-ratio and Stress levels to optimum fatigue test parameters were investigated by using Taguchi technique.

In the same year 2011, Ferit Ficici, and Mesut Durat [29] applied Taguchi design method to study wear behaviour of boronized AISI 1040 steel. AISI 1040 steel was boronized using the packed boronizing technique. Processes were carried out at the temperature of 950°C for 2 and 4 h of treatment. The wear resistance model for AISI 1040 steel was developed in terms of boronizing time, applied load, sliding distance and sliding speed using the Taguchi method.

1-8 The goal of the dissertation

Taguchi method is a problem-solving tool which can improve the performance of the product, process design and system. This method combines the experimental and analytical concepts to determine the most influential parameter on the result response for the significant improvement in the overall performance

The main goal can be divided into the following :

1- Present a new approach of robust technique (Taguchi Methods) based on Mahalanobis Distance ,that is used for validation of the measurement scale.

2- Orthogonal Array (OA and Signal–To noise S/N) ratio values are used for identifying useful features from those under study.

3- Introduce a multidimensional suggested by systems, inspection system, medical diagnosis system and it is important to have a measurement scale by which the degree of abnormality can be measured .

4- The proposed of Taguchi methods MTGS can find application in several areas or fields including industrial, agricultural and medical diagnoses, meaning that the used is limited .

Chapter One
Section Two

Taguchi Methods
Loss Function

Chapter One

Section Two :

Guide to this dissertation

Most of the chapters in this dissertation are developed based on chapters three which provides an introduction to MTS and MTGS methods to get an idea a bout these methods and to modify a method derived from both of them.

1-9 Taguchi Methods

1-9-1- Who is Dr. Genich Taguchi ? [2][6][9][15][17][10][51]

Dr.Geniechi Taguchi was born in Tokamachi Niigata prefecture 120 miles north of Tokyo –Japan in (1924), a city famous for the kimono industry. His father died when he was 10 years old leaving him with a hard working mother, Fusaji and three younger brothers. Tokamachi has been very famous centuries for production of Japanese kimonos.

G. Taguchi attended kiryu Technical College majoring in textile engineering as he was expected to assume responsibility of the family kimono business. But in 1942 Taguchi's draft notice came and with it came an interest in statistics. Under the guidance of Prof. Masuyame, at the time he was regarded by many as the best statistician .Taguchi's statistics skills were nurtured and honed.

In 1962 Taguchi was granted a Ph.D degree in science from (Kyushu University) for his work in developing the industrial design of experiments.

Taguchi found the Quality Research Group. Since 1963 the group has met monthly to discuss industry applications.

In 1965 Taguchi was invited by Aoyame Gakuin University in Japan, he satyed on for 17 years. And he helped develop the university's engineering department.

He observed that a great deal of time and money was expended in engineering experimentation and testing. Little emphasis was given to the process of creative brainstorming to minimize the expenditure of resources.

. In 1982, the American Supplier Institute introduced Taguchi and his methods to the U.S. market. Since that time, companies that have adopted his techniques and philosophy have collectively saved hundreds of millions of dollars worldwide.

In 1986, he received the Willard F. Rockwell Medal for combining engineering and statistical methods to achieve rapid improvements in cost and quality by optimizing product design and manufacturing processes. He received the Blue Ribbon Award from the Emperor of Japan in 1990 for his contribution to industry.

Taguchi started to develop new methods to optimize the process of engineering experimentation..

He believed that the better way to improve quality was to design and build it into the product. Quality improvement starts at the very beginning, i.e., during the design stages of a product or a process, and continues through the production phase. He proposed an "off-line" strategy for developing quality improvement in place of an attempt to inspect quality into a product on the production line. He observed that poor quality cannot be improved by the process of inspection, screening and salvaging. No amount of inspection can put quality back into the product;

Briefly we can say that the Taguchi methods are divided into four periods.

I- The Beginning 1980-1984:[15]

In 1980 Taguchi wrote a letter to Bell Labs indicating that he was interested in visiting them while on sabbatical .He visited there on his own expense and lectured on his methodology.

II- Second period :

Application to Quality Problem Solving 1984 -1992

III- Third Period :

Applications to Product/Process Optimization 1992-2000

V- Fourth Period :

Institutionalization of Robust Engineering 2000-Present

1-9-2 Taguchi Philosophy [1][2][9][14][15][37]

Taguchi espoused an excellent philosophy for quality control in the manufacturing industries. Indeed , his doctrine is creating an entirely new different breed of engineers . He has in fact given birth to a new quality culture in the country. His philosophy has far reaching consequences, The basic of his philosophy is that ,whenever something is produced , a cost is imposed on society. Part of that cost is borne by the producer, and part is borne by the customer as the result of using the product or the service.

1-9-3 Taguchi Methods for Manufacturing Process Control

What is the definition of quality? Responses frequently given include: "conformance to specifications," "zero defects," or "meeting client requirements."

Dr. Taguchi promotes a different, more holistic view of quality. Quality relates to cost and "loss" in dollars, not just to the manufacturer at the time of production, but to the consumer and to society as a whole.

What is "loss" and who pays for it? We usually think of loss as additional manufacturing costs incurred up to the point a product is shipped. After that, it is society, the client, that bears the cost or loss due to poor quality. When the client stops bearing this loss, the manufacturer is out of business.

1-9-4 The basic concepts of Taguchi philosophy [1][9]

The basic of his philosophy depend on the three simple fundamental concepts:

i. **Quality should be designed into a product, not inspected into it.**

Quality is designed into a process through system design, parameter design, and tolerance design. Parameter design, which will be the focus of this article, is performed by determining what process parameters most affect the product and then designing them to give a specified target quality of product. Quality "inspected into" a product means that the product is produced at random quality levels and those too far from the mean are simply thrown out. Therefore, the quality concepts should be based upon, and developed around, the philosophy of prevention. The product design must be so robust that it is immune to the influence of uncontrolled environmental factors on the manufacturing processes. He emphasizes that quality is what one designs into a product.

ii. Quality is best achieved by minimizing the deviation from a target[9].

The product should be designed so that it is immune to uncontrollable environmental factors**.** In other words, the signal (product quality) to noise (uncontrollable factors) ratio should be high. His second concept deals with actual methods of effecting quality. He contended that quality is directly related to deviation of a design parameter from the target value, not to conformance to some fixed specifications. A product may be produced with properties skewed towards one end of an acceptance range yet show shorter life expectancy. However, by specifying a target value for the critical property and developing manufacturing processes to meet the target value with little deviation, the life expectancy may be much improved.

iii. The cost of quality should be measured as a function of deviation from the standard and the losses should be measured system wide.

This is the concept of the loss function, or the overall loss incurred upon the customer and society from a product of poor quality. Because the producer is also a member of society and because customer dissatisfaction will discourage future patronage, this cost of customer and society will come back to the producer.

And his third concept calls for measuring deviations from a given design parameter in terms of the overall life cycle costs of the product. These costs would include the cost of scrap, rework, inspection, returns, warranty service calls and/or product replacement. These costs provide guidance regarding the major parameters to be controlled.

Taguchi views quality improvement as an ongoing effort. He continually strives to reduce variation around the target value. A product under investigation may exhibit a distribution that has a mean value different from the target value. The first step towards improving quality is to achieve the population distribution as close to the target value as possible. To accomplish this objective Taguchi designs experiments using especially constructed tables known as "orthogonal arrays" (OA). The use of these tables makes the design of experiments very easy and consistent.

A second objective of manufacturing products to conform to an ideal value is to reduce the variation or scatter around the target.

Taguchi cleverly incorporates a unique way to treat noise factors. Noise factors, according to his terminology, are factors which influence the response of a process, but cannot be economically controlled. He devised an effective way to study their influence with the least number of repetitions. The end result is a "robust" design affected minimally by noise, with a high signal to noise value.

Taguchi proposes a holistic view of quality related to cost, he defines quality as the (minimum) loss imparted by the product to society from the time the product is shipped . His methods provide an efficient and symmetric way to optimize designs for performance, quality and cost , he breaks down off-line quality control into three stages, system design , parameter design, and tolerance design, which are summarized .

To achieve desirable product quality by design, Taguchi recommends three stage processes.

1- Systems design

2- Parameter design

3- Tolerance design

1- System design result in either a design concept or and “up and limping“ prototype, in the initial phase, more than one design concept. The ideal design system will be the one that research shows best addresses customer needs and is inherently robust. The purpose of system design procedure is to find the suitable working levels of the design factors

2- Parameter design is a critical production step. The nominal design features or selected process factor levels are tested and combination of product parameter levels or process operating levels least sensitive to changes in environmental conditions and other uncontrolled factors (noise) is determined . [31]

The parameter design procedure determines the factor levels that can generate the best performance of the product or process under study

3-The tolerance design is used to further reduce variation, if required, by tightening the tolerance of those factors shown to have a significant impact on variation. This stage utilizes loss function to determine whether spending more money on materials and equipments will result in a better product.[31] .

The tolerance design procedure is used to fine-tune the results of parameter design by tightening the tolerance levels of factors that have significant effects on the product or process

The goal of this point is to minimize variations around a target value.

Taguchi’s philosophy is based on the belief that once quality is designed into both the product and process, only minimal inspection is necessary.

And the Taguchi idea can be explained by some points such as [49]:

i. Measure the degree of unhealthiness or abnormality of different conditions by introducing measurement scale based on input characteristics

ii. Identify the direction of abnormal conditions

iii.Minimize the number of variables required for an effective diagnosis, to establish different zones of treatment of product.

Taguchi contribution has also made the work of the practitioner simpler by advocating the use of less experimental designs and by providing a clearer understanding of the natural variation of various parameters.

In general, the parameter design of the Taguchi method utilizes orthogonal arrays (OAs) to minimize the time and cost of experiments in analyzing all the factors and uses ANOVA (analyses of variance) and the signal-to-noise (S/N) ratio to analyze the experimental data and find the optimal parameter combination. Using OAs significantly reduces the number of experimental configurations to be studied. Taguchi's method separates factors into three categories: control factors, which are important in reducing variation; adjustment factors, which are used to set output at a desired target; and cost-adjustment factors, which, though unimportant for determining variation of output levels, are useful for improving a product's cost effectiveness.

Figures (1-3-a),(1-3-b),(1-3-c),and (1-3-d) show that the discretion of the main idea about the Taguchi Method where the input signal is the actual or the true value of the state of the system, and with the different types of noise from different ways should match closely. $Variables(X_1, X_2,..Xn)$

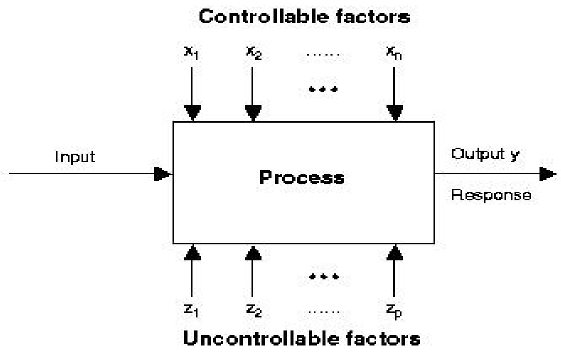
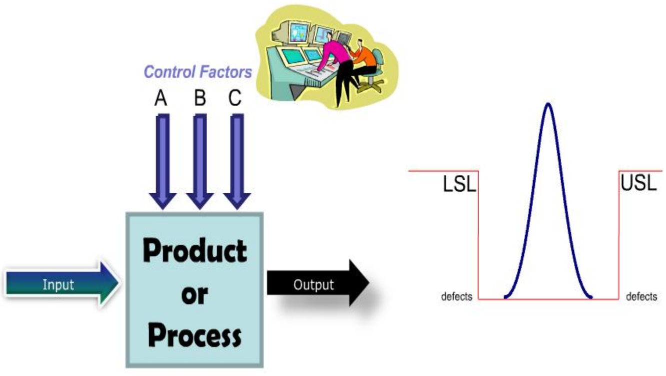


Fig.(1-3-a) Flow chart of Taguchi Method

Robust Design



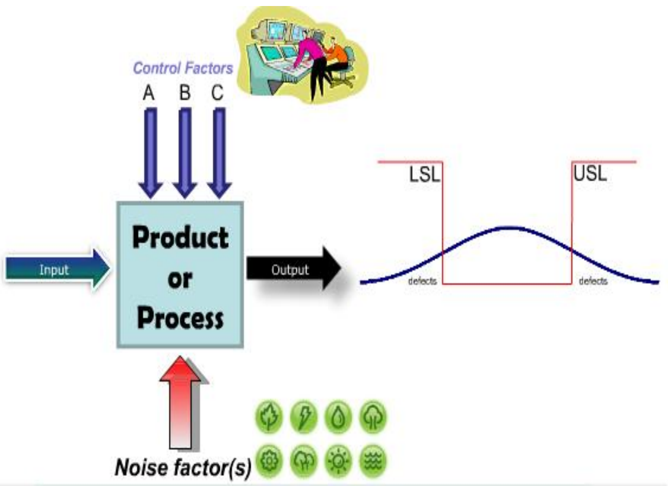


Fig.(1-3-b) Flow chart of Taguchi Method

Fig.(1-3-c)Flow chart of Taguchi Method [18]

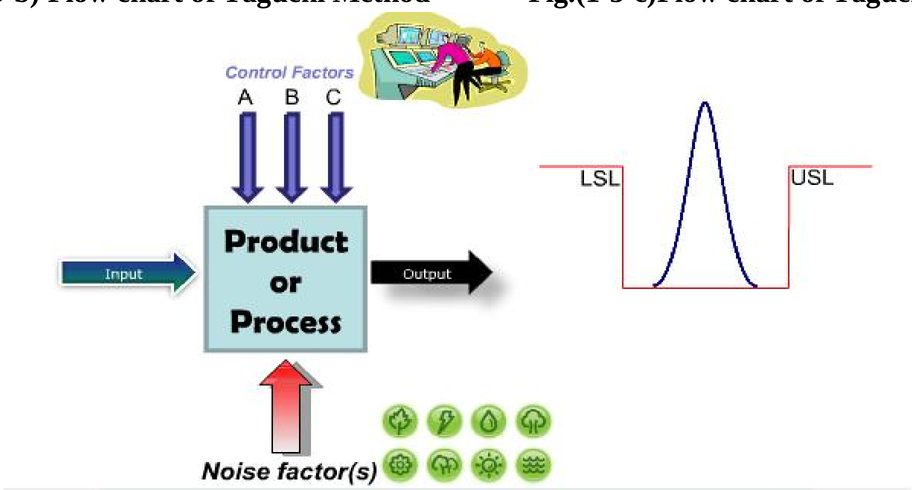


Fig.(1-3-d) Flow chart of Taguchi Methods [18]

1-9-5 Assumptions of the Taguchi method [36]

The additive assumption implies that the individual or main effects of the independent variables on performance parameter are separable. Under this assumption, the effect of each factor can be linear, quadratic or of higher order, but the model assumes that there exists no cross product effects (interactions) among the individual factors. That means the effect of independent variable 1 on performance parameter does not depend on the different level settings of any other independent variables and vice versa. If anytime, this assumption is violated, then the additively of the main effects does not hold, and the variables interact.

1-10 Taguchi's Robust Design Method [2][12][16]

The concepts of robust engineering (RE) are based on the philosophy of Taguchi, who introduced the concept after several years of research. Robust engineering systematically evolved starting in the 1950.

Since 1960, Taguchi methods have been used for improving the quality of Japanese products with great success. During the 1980's, many companies finally realized that the old methods for ensuring quality were not competitive with the Japanese methods.

The old methods for quality assurance relied heavily upon inspecting products as they rolled off the production line and rejecting those products that did not fall within a certain acceptance range.

However, Taguchi was quick to point out that no amount of inspection can improve a product; quality must be designed into a product from the start. It is only recently that companies in the United States and Europe began adopting Taguchi's robust design approaches in an effort to improve product quality and design robustness.

Taguchi started to develop new methods to optimize the process of engineering experimentation. He developed techniques which are now known as the Taguchi Methods.

His greatest contribution lies not in the mathematical formulation of the design of experiments, but rather in the accompanying philosophy, so the Robust design is an engineering methodology for improving productivity during research and development so that high-quality products can be produced quickly and at low cost, and a design that has minimum sensitivity to variations in uncontrollable factors.

A key philosophy of robust design is that during the optimization phase, inexpensive parameters can be identified and studied, and can be combined in a way that will result in performance that is insensitive to noise. The team's task is to determine the combined best settings (parameter targets) for each design parameter, which have been judged by the design team to have potential to improve the system.

By varying the parameter target levels in the transfer function (design point), a region of nonlinearity can be identified. This area of nonlinearity is the most optimized setting for the parameter under study.

Consider two levels or means of a design parameter (DP), level 1(DP') and level 2 (DP"), having the same variance and distribution. It is obvious that level 2 produces less variation in the FR than level 1.

Level 2 will also produce a lower quality loss, The design produced by level2 is more robust than that produced by level 1 as shown in fig(1-4).[16]

Level 2 will also produce a lower quality loss similar to the scenario at the bottom of Fig(1-5). The design produced by level 2 is more robust than that produced by level 1.

When the distance between the specification limits is 6 times the standard deviation, a Six Sigma optimized FR is achieved. When all design FRs are released at this level, a Six Sigma design is achieved.

Taguchi has defined a number of methods to simultaneously reduce costs and improve quality.

The popularity of his approach is a fitting testimony to the merits of this work.

The Taguchi methods may be considered under four main headings:

- total loss function;
- design of products, processes and production;
- reduction in variation;
- statistically planned experiments.

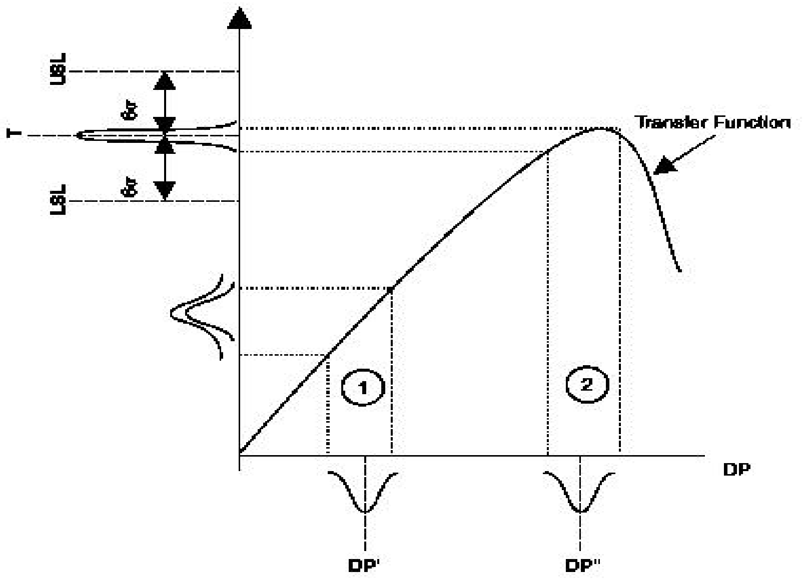
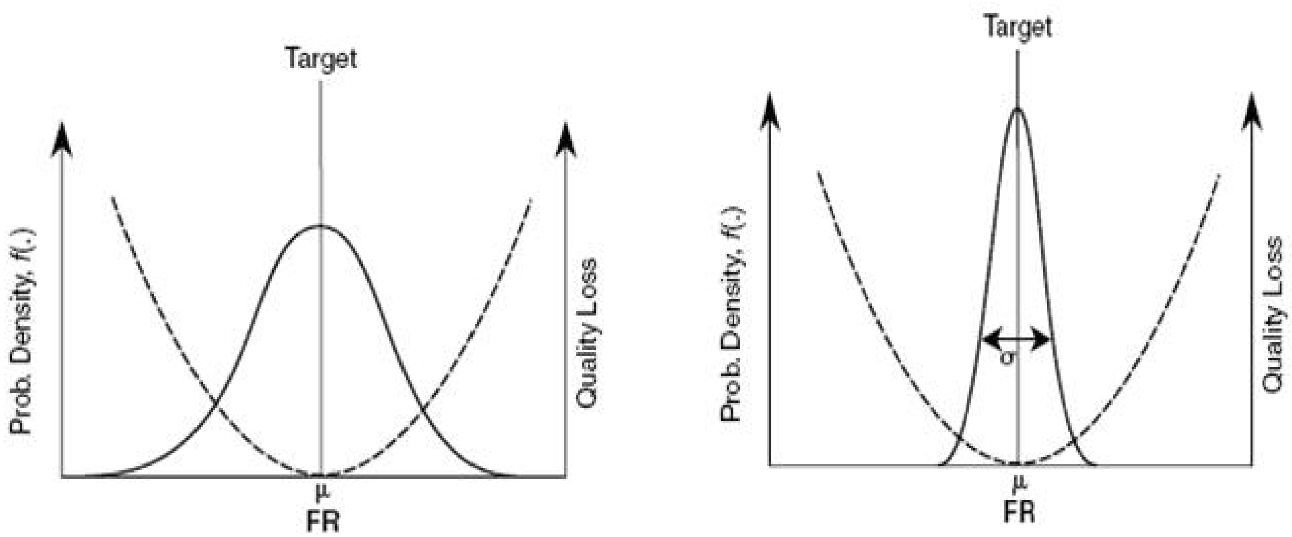


Fig.(1-4)Transfer function and optimization [16]

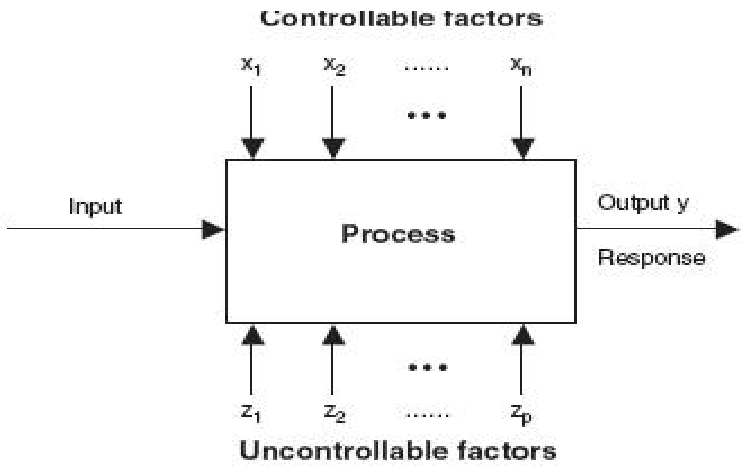


Figure(1-5)The quality loss function. [16]

1-11 Introduction to Design of Experiments (DOE) [4][6][16]

Design of experiments is also called statistically designed experiments. The purpose of the experiment and data analysis is to find the cause and-effect relationship between the output and experimental factors in a process.

The process model of DOE is illustrated in Fig. (1-6)



Fig(1-6) A process model of DOF [16]

Mathematically, the researcher is trying to find the following functional relationship:

$$y = f(x_1, x_2, x_3, \ldots x_n) + \varepsilon \qquad \ldots 1\text{-}1$$

where (ε) is experimental error, or experimental variation. The existence of (ε) means that there may not be an exact functional relationship between *y* and ($x_1, x_2, \ldots, x_n$). This is because :

1. Uncontrollable factors ($z_1, z_2, \ldots, z_p$) will influence the response y but are not accounted for in Eq. (1-1).
2. There are experimental and measurement errors on both y and ($x_1, x_2, \ldots, x_n$) in the experiment.

1-12 Loss Function [3][5[9][15][16]

1-12-1 Introduction

The basic idea of Taguchi's philosophy is that whenever some things is produced, a cost ? is imposed on society, it comes by parts; one of them comes from the producer and another part is borne by the customer as the result of using the product or service, then costs are designed or plotted by the function of quality, producer costs tend to increase with increased quality the customer costs tend to decrease because of greater efficiency.

Loss is usually thought of as additional manufacturing costs incurred up to the point that a product is shipped .

1-12-2 Definition of Genichi Taguchi 1986 for loss function:[32]

"An article of good quality performs its intended functions without variability , and causes little loss through harmful side effects, including the cost of suing it."

Then :

i- The key point is without variability. Prior to the ideas of Taguchi, people thought the production was okay as long as the products were within the tolerance. The reduction of variability is the primer goal of Taguchi quality control.

ii- To enhance the quality, we want to reduce the loss

iii- Modeling the loss by quality loss function is central.

Taguchi defined the loss function as deviation as a quantity proportional to the deviation from the target quality characteristic. At zero deviation, the performance is on target, the loss is zero. If Y represents the deviation from the target value, then the loss function.

The loss function is calculated from the square of the reciprocal of the process capability index after multiplying a constant related to economy. It is an economic forecasting value that is imparted to the customer in the market.

The process capability index, Cp, is calculated by the following equation:

$$C_p = \frac{tolerance}{6(s\tan dard\ deviation)} \qquad \ldots 1\text{-}2$$

The loss function is given as

$$L(y) = k(y-m)^2 \qquad \ldots 1\text{-}3$$

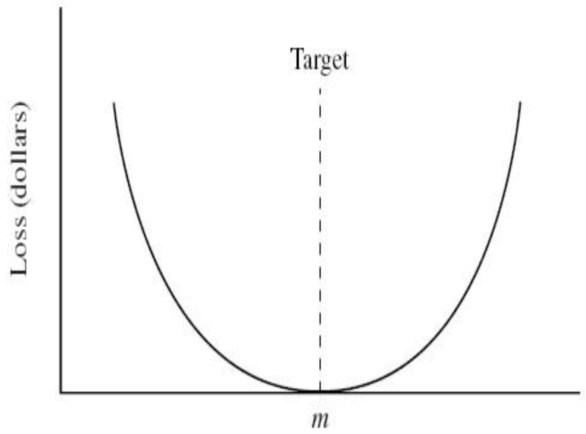
Where

L =loss function

y= The quality characteristics, such as a dimension performance, or output value

m = The target value for the quality characteristics.

k = is a constant dependent upon the cost structure of manufacturing process.



Figure(1-7) Quality loss function [taguch]

From loss function figure (1-7) it shows that:

a- The loss function must be equal zero when the quality characteristic of the product meets its target value.

b- The magnitude of the loss function increases rapidly as the quality characteristics deviates from the target value .

c- The loss function must be continue (second order) as function of the deviation from the target value. Which is to be minimized, where (y) is the quality characteristic, (m) is the target value and (k) is an appropriate constant. He also uses signal-to-noise (SN) ratios which are based in principle on the loss function.

Taguchi determined the loss function from a Taylor Theorem expansion about the target value (m).

The loss function is given as :

$$L(y) = k(y-m)^2 \qquad \text{...1-4}$$

Thus:

$$L(y) = L(m) + \frac{L^{'}(m)}{1!}(y-m) + \frac{L^{''}(m)}{2!}(y-m)^2 + higherorde\ rterms \qquad \text{.... 1-5}$$

Because L is minimum at $y = m$, $L^{'}(m) = 0$, $L = (m)$ is always a constant and is ignored since its effect is to raise or lower the value of $L(y)$ uniformly at all values of y. The $(y-m)^2$ term is dominant term in equation (1-5) (higher – order term are neglected. There for we use the following equation as in approximation:

$$L(y) = L(m) + \frac{L^{''}(m)}{2!}(y-m)^2 \qquad \text{... 1-6}$$

Then the expression $\frac{1}{2!}L^{''}(m)$ in Equation (1-6) is constant as(k) therefor the equation (1-6) can be written as :

$$L(y) = k(y-m)^2 \qquad \text{....1-7}$$

For the product with a value (m) target from most customer point of view $m \pm \Delta_0$ represents the deviation at which functional failure of product or component occurs. When a product is manufactured with its quality characteristic at the extremes .

$m + \Delta_0$ or at $m - \Delta_0$, some countermeasure must be undertaken by the average customer m: target value for a critical product

$+/-\Delta_0$: allowed deviation from the target

A_0: loss due to a defective product

$$L(y) = k(y-m)^2 \qquad \text{....1-8}$$

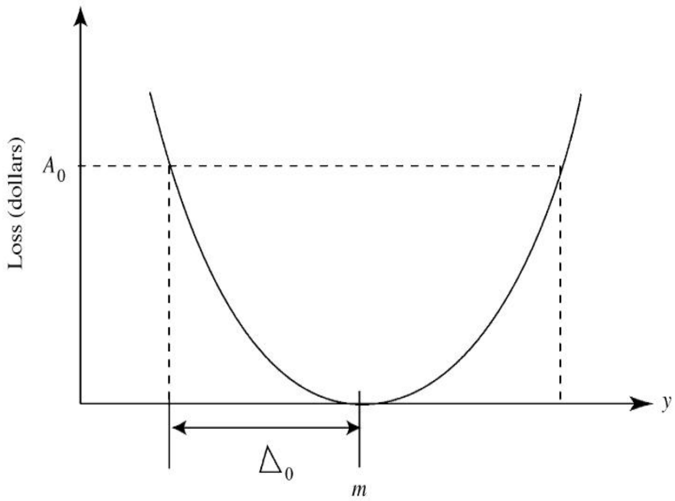
At $y = m + \Delta_0$

$$A_0 = k(m+\Delta_0 - m)^2 \qquad \text{...1-9}$$

Now the constant k can be solved:

$$k = \frac{Ao}{\Delta_0^2}$$

This value of k which constant for a given quality characteristic, and the target value of m ,completely define the quality loss function curve as shows in Fig (1-8):



Figure(1-8) Quality loss function

Up to this point the quality loss function is explained with one piece of product .

In general , there are multiple pieces of product .In that case the loss function is given by :

$$L = k\delta^2 \qquad \text{...1-10}$$

Where :

$$C_p = \frac{Tolerance}{6\,(s\tan dard\ deviation)}$$

There for can be the equation C_p written as

$$Cp = \frac{2\Delta_0}{6\sigma}$$

$$L = \frac{Ao}{\Delta_0^2}\sigma^2 = \frac{Ao}{9Cp^2}$$

It is seen that the loss function can be expressed mathematically by the process capability index.

1-12-3 Classification of Quality Characteristics: [15][16][21][40]

Quality loss function is used for :

1-Nominal-the best Characteristics

2-Smaller –the –better Characteristics

3-Larger –the-better Characteristics

i-Quality Loss function for Nominal-the-best quality characteristics

The Nominal –the best **characteristics** is the type where there is a finite target point to achieve. There are typically upper and lower specification limits on both sides of the target.

$$L(y) = k(y-m)^2 \qquad \text{…1-11}$$

The loss function, L as described previously, is used for evaluating one unite of product:

Where Mean Squared Deviation (MSD)

$$MSD = (y-m)^2 \qquad \text{….1-12}$$

$$L(y) = k(MSD) \qquad \text{….1-13}$$

For a group of n-product with output $y_1, y_2, y_3, \dots y_n$ the average loss is

$$L(y)=\frac{k(y_1-m)^2+k(y_2-m)^2+....+k(y_n-m)^2}{n}$$

$$MSD=\frac{1}{n}\sum_{i=1}^{n}(y_i-m)^2$$

$$=\frac{1}{n}\sum_{i=1}^{n}(y_i-\bar{y})^2+(\bar{y}-m)^2$$

$$=\frac{(y_1-\bar{y})^2+\dots+(y_n-\bar{y})^2}{n}+(\bar{y}-m)^2$$

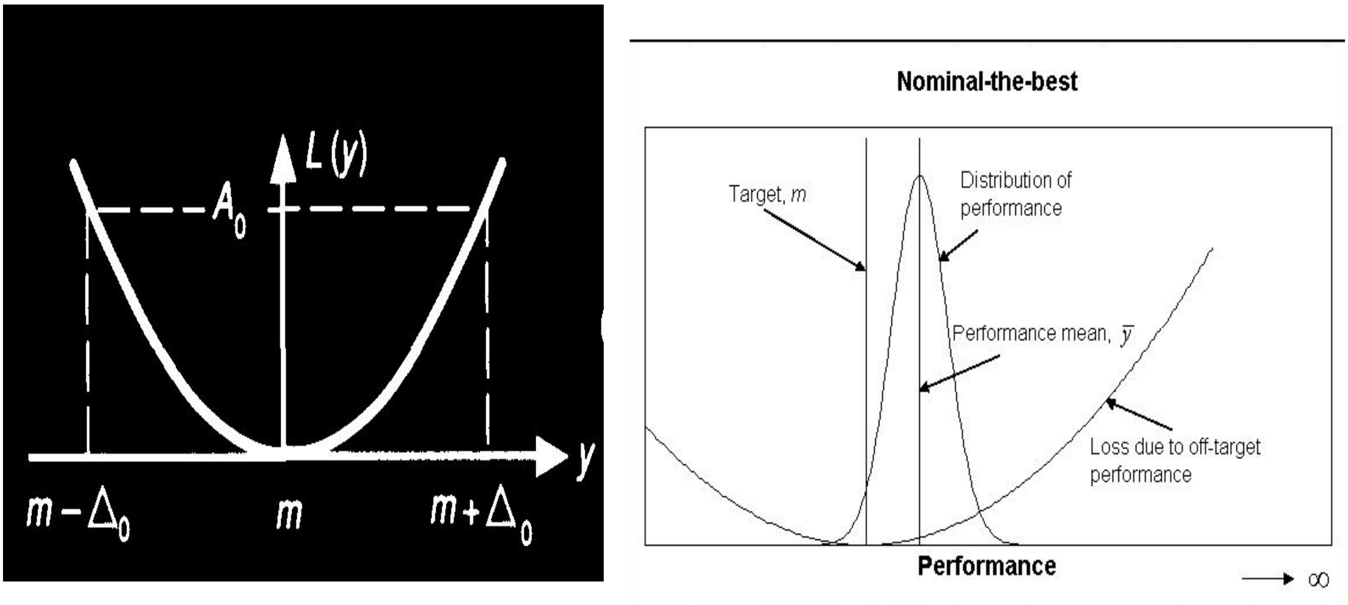
$$=\sigma^2+(\bar{y}-m)$$

Where $\bar{y}$ is the average of y, the loss function for more than one piece then becomes.

$$L(y)=k\left[\sigma^2+(\bar{y}-m)^2\right] \qquad \text{... 1-14}$$

Where (k) is a proportionality constant and y is the output

A graphical representation of the Nominal Characteristic is shown below. As the output value (y) deviates from the target value (m) increasing the mean squared deviation, the loss (L) increases. There is no loss when the output value is equal to the target value (y = m).Fig(1-9) shows the graphical Nrominal chart



Figure(1-9)Nominal chart [40]

ii- A Quality Loss function for Smaller–the –better Characteristic

A smaller –the –better output response is the type where it is desired to minimize the result , with the ideal target being zero .

The equation used to describe the loss function of one unit of product:

$$L(y) = ky^2$$

Where:

k= Proportionality Constant

y = Output Value

The proportionality constant (k) for the Smaller-the-Better characteristic can be determined as follows:

$$k = \frac{A_0}{y^2}$$

$$L(y) = k(MSD)$$

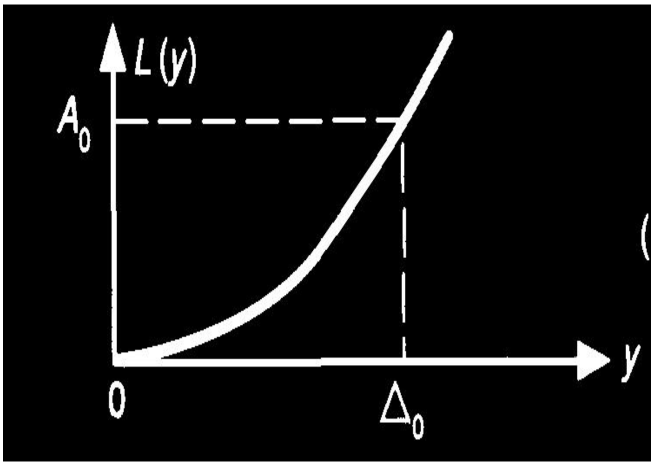
$$MSD = \sum_{i=1}^{n} \frac{y_i^2}{n} = \sigma^2 + \bar{y}^2 \qquad \text{... 1-15}$$

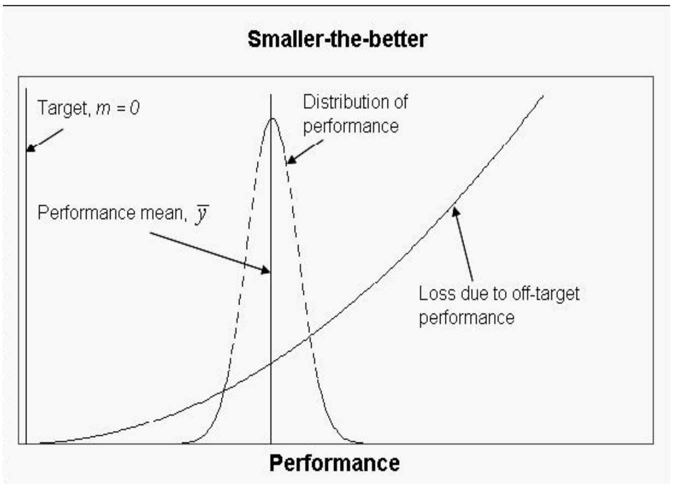
A_0 = Consumer Loss (in Unit)

y = Maximum Consumer Tolerated Output Value

A graphical representation of the Smaller-the-Better characteristic is below.

The loss is minimized as the output value is minimized.Fig(1-10) shows the graphical of smaller-the -Better





Figure(1-10) flowchart of Smaller –the –better [40]

iii-A Quality Loss function for The larger–the –better characteristic

The larger –the –better characteristic output response is the type where it is desired to maximize the result, the ideal target being infinity.

The Larger–the-Better characteristic is just the opposite of the Smaller-the-Better characteristic. For this characteristic type, it is preferred to maximize the result, and the ideal target value is infinity. An example of this characteristic is maximizing the product yield from a process. The equation used to describe the loss function of one unit of product is

$$L(y) = \frac{k}{y^2} ..$$

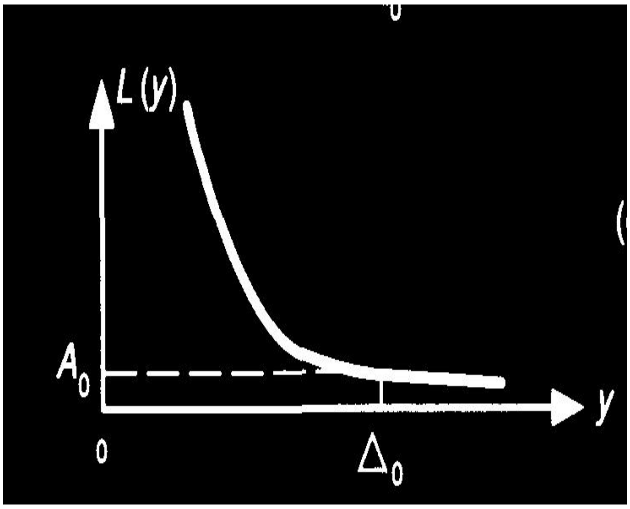
Where:

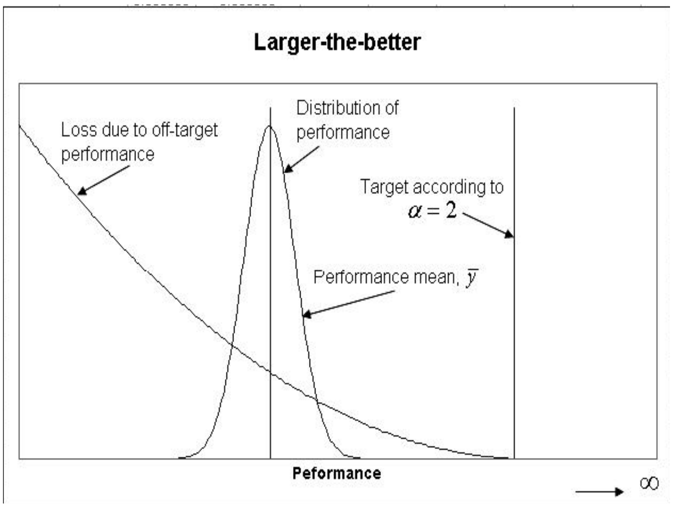
k = Proportionality Constant

y = Minimum Consumer Tolerated Output Value

$$k = A_0 y^2$$
$$L(y) = k(MSD) \qquad ... 1\text{-}16$$
$$MSD = \frac{1}{n}\sum \frac{1}{y_i^2}$$

The proportionality constant (k) for the Large-the-Better characteristic can be calculated by using the equation given for the Smaller-the-Better proportionality constant. The only difference between the two is the definition of y.





Figure(1-11) flowchart of Larger–the –better[40]

Chapter One
Section Three

Mahalanobis Distance
Introduction To Taguchi
MTS, MTGS and S/N Ratio

Chapter One

Section Three

Mahalanobis Distance MD

This Chapter introduces the methodologies of the MD-MTS and MD-MTGS Basic steps in these methods are described .

We begin with a detailed discussion of Mahalanobis distance.

1-13 Introduction [15][21][22][49]

The Mahalanobis – Prasanta Chandra Mahalanobis (P.C.Mahalanobis)
He was bourn in Calcutta, India at 29-jun-1893 and died in 28-jun-1972 Calcutta - India at the Age of 78 .

• He was a famous Indian statistician who established the (Indian Statistical Institute (ISI) in 1936,which is very influential in large-scale sample survey methods. He was an architect of India's industrial strategy, and an advisor to Nehru and friend of R.A. Fisher. He introduced a statistical tool called Mahalanobis Distance MD, in other words, Mahalanobis Distance is a process of distinguishing one group from another. [13]

The Mahalanobis is a generalized distance, which can be considered a signal measure of degree of divergence in the mean values of different characteristics of population by considering the correlations between the variables, Mahalanobis distance measures distances in multidimensional spaces by taking correlations into account.

Mahalanobis distance is a very useful way of determining the similarity of a set of values from an unknown sample to aset of values measured from a collection of known samples. MD is used to class the observation into different groups.

The observation is classified into a group from whose center it has the least distance.

The main reasons for using MD is very sensitive to the correlation structure of the reference group.

MD used to find the “nearness” of unknown points from the mean point of a group as shown in Fig(1-12).

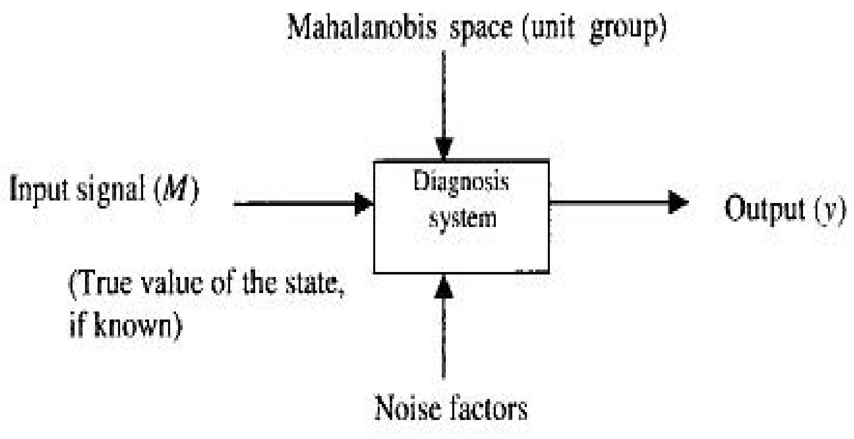


Figure (1-12) Modified multidimensional diagnosis distance [14]

There are other multivariate measurement techniques, such as Euclidean distance (ED). Euclidean distance also gives the distance of the “unknown’ point from the group mean point, but there are two disadvantages of this technique ;

i- Euclidean distance does not give a statistical measurement of how well the unknown matches the reference set.

ii- It measures only a relative distance from the mean of point in the group and does not take into account the distribution of the points in the group.

Fig.(1-13) shows that a comparison between Mahalanobis distance (MD) and Euclidean distance (ED). This will be clear if we compute MD instead of (ED), because the (ED) method does not take into account the correlations between variables, therefor ED is less helpful than MD.

Mahalanobis space (MS) is a database containing the means, standard deviations ,and correlation structure of the variables in the reference group which has average unit distance . And MS has the zero point at its mean with an average MD equal to unity .

And one can say that the MS, reference group is obtained using the standardized variables of the healthy or normal group data [14] .

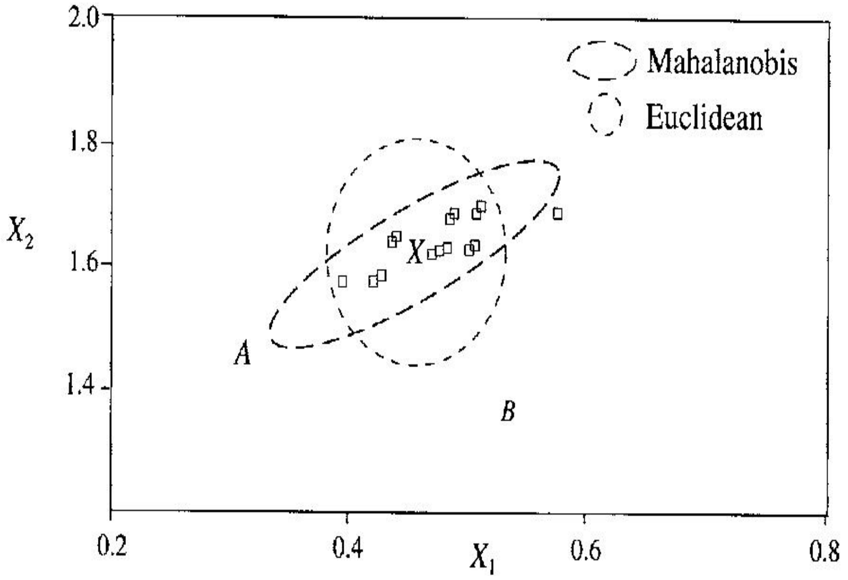


Figure (1-13) Mahalanobis and Euclidean distance [14]

1-13-1 Mahalanobis Distance (Inverse Matrix Methods)

Taguchi introduced the Mahalanobis Distance into the process of defining a reference group and measuring individual subsets MTS was developed by Taguchi as a diagnostic and for casting using multivariate data .

The main objective of his application was to make statistical judgments to distinguish one group from another. The Mahalanobis Distance is a squared distance (D^2) calculated for the i^{th} observation in as ample of size (n) with (k) variables as the following formula:

$$MDj = D_j^2 = Z_{ij}^T C^{-1} Z_{ij} \qquad \ldots 1\text{-}17$$

Where :

j=1 to n

$$Z_{ij} = (z_{1j}, z_{2j}, \ldots.. z_{kj})$$

Z_{ij} = standardized vector obtained by standardizing values of Xij

For i= 1 ,….,k

$$Z_{ij} = \frac{(X_{ij} - \overline{X} j)}{S_j}$$

X_{ij} = Value of the ith characteristic in the jth observation

$\overline{X}_i$ = Mean of the ith characteristic

S_i = Standard deviation (SD) of the ith characteristic

C^{-1} =Inverse of the correlation matrix

1-13-2 The Mahalanobis-Taguchi System (MTS) [15][22]

In the MTS , the MD obtained from equation (1-17) is scaled by dividing in to the number of variables k .Therefor the equation (1-18) becomes as :

$$MDj = \frac{1}{k} Z_{ij}^{T} C^{-1} Z_{ij} \qquad \text{...1-18}$$

$$MD = \frac{1}{k} (z_1, z_2, ... z_{12})(C^{-1}) \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_k \end{pmatrix} \qquad \text{...1-19}$$

Where Z_i is a Vectors of $(Z_1, Z_2, Z_3, ...Z_k)$

And C^{-1} is inverse matrix of correlation matrix

1-13-3 Gram-Schmidt Orthgonalization Process [14][19][41]

MD can also be computed by using Gram-Schmidte Process. which is discretion as follows:

Given linearly Independent Vectors $(Z_1, Z_2, Z_3, ...Z_k)$ there exist mutually perpendicular vectors $(U_1, U_2, U_3, ...U_k)$ with the same linear span

Then the Gram – Schmidt vectors are constructed sequentially by setting:

$$U_1 = Z_1$$

$$U_2 = Z_2 - \frac{Z_2' U_1}{U_1' U_1} U_1$$

$$\vdots \qquad \text{...1-20}$$

$$U_k = Z_k - \frac{Z_k' U_1}{U_1' U_1} U_1 - ..... \frac{Z_k' U_{k-1}}{U_{k-1}' U_{k-1}} U_{k-1}$$

When MD is calculated using Gram-Schmidt Process (GSP), standardized value of variables are used. Therefor in the above set of equation $(Z_1, Z_2, Z_3, \ldots Z_k)$ corresponds to standardized values. It is clear that the transformation process largely depends on the first variables.

Calculating of MD using the Gram-Schmidt Process (GSP):

There is the sample of size (n) and each sample contains observations on (k) variables. After standardizing the variables, we will have a set of standardized vectors:.

$$Z_1 = (z_{11}, z_{12}, \ldots z_{1n})$$

$$Z_2 = (z_{21}, z_{22}, \ldots z_{2n})$$

$$Z_3 = (z_{31}, z_{32}, \ldots z_{3n})$$

M

$$Z_k = (z_{k1}, z_{k2}, \ldots z_{kn})$$

Where Z_1, Z_2,….Z_k are the set of standardized vectors.

And the GSP orthogonal vector as:

$Z_1, Z_2, Z_3, \ldots Z_k$ → GSP → $U_1, U_2, U_3, \ldots U_k$ Orthogonal Vectors

Figure (1-14) Gram –Schmidt process

Where

$$U_1 = (u_{11}, u_{12}, \ldots u_{1n})$$

$$U_2 = (u_{11}, u_{22}, \ldots u_{2n})$$

M

$$U_k = (u_{k1}, u_{k2}, \ldots u_{kn})$$

The mean of vectors $(U_1, U_2, U_3, \ldots U_k)$ are zero and with $(S_1, S_2, S_3, \ldots S_k)$ respectively be the standard deviation of $(U_1, U_2, U_3, \ldots U_k)$ for sample size n , there will be (n) MDs, and the MD is calculated by the following :

$$MD_j = \frac{1}{k}\left(\frac{u_{1j}^2}{s_1^2} + \frac{u_{2j}^2}{s_2^2} + \frac{u_{3j}^2}{s_3^2} + \cdots + \frac{u_{kj}^2}{s_k^2}\right) \qquad \ldots 1\text{-}21$$

Where j= 1, …,n , the values of MD obtained from (1-18) and (1-21) are exactly the same .

To prove that The equation (1-18) and (1-21) are the same it is known that the Gram –Schmidt vectors $(U_1, U_2, U_3, \ldots U_k)$ are mutually perpendicular vectors, and $(S_1, S_2, S_3, \ldots S_k)$ are be the standard deviation of $(U_1, U_2, U_3, \ldots U_k)$ these vectors are obtained from standardized vectors $(Z_1, Z_2, Z_3, \ldots Z_k)$ and the mean of these vectors are equal to zero ,

There is

$$MDj = \frac{1}{k} U_{ij}^T C^{-1} U_{ij} \qquad \ldots 1\text{-}22$$

where

$$C = \begin{bmatrix} S_1^2 & 0 & \cdots & 0 \\ 0 & S_2^2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & S_k^2 \end{bmatrix}$$

Since this is a diagonal matrix

$$C^{-1} = \begin{bmatrix} \frac{1}{S_1^2} & 0 & \cdots & 0 \\ 0 & \frac{1}{S_2^2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \frac{1}{S_k^2} \end{bmatrix}$$

and $U_{ij}' = (u_{1j}, u_{2j}, ,,,, u_{kj})$

where j=1,…n

Therefor after performing the matrix multiplication

$$MD_j = \frac{1}{k}\left(\frac{u_{1j}^2}{s_1^2} + \frac{u_{2j}^2}{s_2^2} + \frac{u_{3j}^2}{s_3^2} + \cdots + \frac{u_{kj}^2}{s_k^2}\right) \qquad \ldots 1\text{-}23$$

1-13-4 Calculating the mean of the Mahalanobis Space MS [14]

It has been proved that MD (without scaling) follows a Chi–Squr(χ^2) distribution with k degrees of freedom , when the sample size n is large and all the characteristics follow the normal distribution .

This proof can be obtained from (Johnson and Wichern (1992)).

It is know that (χ^2) statistic with k degree of freedom has a mean equal to k .

Hence, scaled MD has a mean of (1.0)

In fact, the assumptions of distribution of input variables are not necessary for calculating MDs in MS. This is proved in the following steps:

Proof that MD can be obtained without assumption of distribution of variables [15]

Let Xj= (j= 1,2,…n) be a variable with mean μ and standard deviation (σ.)

The quality $\frac{(X_j - \mu)}{\sigma}$ known as the standardized variable, measures the distance of X_j from (μ) in terms of SD unite .

The quantity $\frac{(X_j - \mu)^2}{\sigma^2}$ measured the squared distance of X_j from (μ) in terms of SD units. The measurement scale for such quantities will start from the zero and all the measured distances are positive.

It can be easily shown that the expected value of $\frac{(X_j - \mu)^2}{\sigma^2}$ is equal to (1)

This can be mathematically written as . .

$$E\left[\frac{(X_j - \mu)^2}{\sigma^2}\right] = 1 \qquad \text{…1-24}$$

And the equation can be written as:

the quantity as $\frac{(X_j - \mu)^2}{\sigma^2}$ it can be written as :

Squared standardized distance = $(X_j - \mu)(\mathrm{var}\,iance)^{-1}(X_j - \mu)$ ….1-25

From Equation (1-18) there is $MD_j = \frac{Z_{ij}^T C^{-1} Z_{ij}}{k}$ similar as where i= 1,2,…k

Where k is number of variable characteristics

Eq.(1-18) and Eq.(1-25) are similar because the in Eq.(1-25) measures squared distance in the univariate case and Eq.(1-18) measures squared distance in multivariate case . In the multivariate case , since one has to take correlation into account , the correlation matrix replaces the variance term.

MD can also be represented in terms of Gram-Schmidt variables using Eq(1-23) which is given as :

$$MD_j = \frac{1}{k}\left(\frac{u_{1j}^2}{s_1^2} + \frac{u_{2j}^2}{s_2^2} + \frac{u_{3j}^2}{s_3^2} + \cdots + \frac{u_{kj}^2}{s_k^2}\right) \qquad \ldots 1\text{-}26$$

In this equation $u's$ are the Gram-Schmidt variables, and $(S_1, S_2, S_3, \ldots S_k)$ are the standard deviation of these variables ,since the Gram - Schmidt process is carried out with the help of standardized variables, the means of $u_{1j}, u_{2j}, .. u_{kj}$ are zeros The squared terms of Eq.(1-26) correspond to the squared distance of Gram-Schmide value of squared distances in Eq.(1-26), it is equal to Eq.(1-24).

Now

$$E(MD_j) = E\left[\frac{1}{k}\left(\frac{u_{1j}^2}{s_1^2} + \frac{u_{2j}^2}{s_2^2} + \frac{u_{3j}^2}{s_3^2} + \cdots + \frac{u_{kj}^2}{s_k^2}\right)\right] \qquad \ldots .1\text{-}27$$

or

$$E(MD_j) = (\frac{1}{k})\left[E\left(\frac{u_{1j}^2}{s_1^2} + \frac{u_{2j}^2}{s_2^2} + \frac{u_{3j}^2}{s_3^2} + \cdots + \frac{u_{kj}^2}{s_k^2}\right)\right] \qquad \ldots .1\text{-}28$$

Since Gram Schmid’s vectors are orthogonal ,one can writ the equation as

$$E(MD_j) = (\frac{1}{k})\left[E\left[\frac{u_{1j}^2}{s_1^2}\right] + E\left[\frac{u_{1j}^2}{s_2^2}\right] + \ldots . + E\left[\frac{u_{kj}^2}{s_k^2}\right]\right] \qquad \ldots .1\text{-}29$$

Since expected values on the right-hand side are equal to one, we have

$$E(MD_j) = \left(\frac{1}{k}\right)\frac{[1+1+\cdots+1]}{k\,times}$$

$$E(MD_j) = \left(\frac{1}{k}\right)(k) = 1$$

Therefore the expected MD has properties of the quantity $\frac{(X_j - \mu)^2}{\sigma^2}$

because it measures distances from the zero point and has an expected value equal to unity.

With this proof, one can say that we do not require the assumption of any distribution to define the zero point and unit distance and hence it can be generalized for any number of variables irrespective of their distribution . MS is centered at zero point because the original variables are converted into standardized variables .One can not define the zero point and unit distance if the standardized variables are not used and the scaling is not done.[14]

Then advantages of defining the Mahalanobise Space MS with scaled MDs areas follows:

i- The definition of MS can be generalized to any number of variables.

ii- The average of value of MDs in MS is always equal to (1).

1-14 Signal –to –Noise Ratio [10][16][24][27]

1-14-1 Introduction

The signal-to-noise ratio (S/N ratio) is a measurement scale that has been used in the communication industry for nearly a century.

Taguchi has generalized the concept of SN ratio as used in the communication industry and applied it for the evaluation of measurement systems as well as for the function of products and processes.

The Taguchi approach to experimental design utilizes what are termed signal-to-noise ratios. As discussed by Kacker (1985), Taguchi has defined more than 60 signal-to-noise ratios (S/N) for various engineering applications.

The general idea is to use an (S/N) that is appropriate for a particular situation. The S/N, which is to be maximized, is presumed to be a logical estimator of some performance measure, and to minimize the number of variables.

In both methods, (MTS and MTGS), the S/N ratio (measure of accuracy of the measurement scale) is used to identify the useful variables .S/N ratio are used as a basis of a “Larger-the better” criterion. Larger–the–better–type S/N ratios are used because procedures used then in both methods **.**

The signal–to–noise ratio (S/N) is a measurement scale that has been used in the communication industry for nearly a century. The S/N ratio is used in the communication industry and applied for the evaluation of measurement system as well as for the function of product and processes.

In communication engineering, Fig.(1-15) is a typical representation of how the system works. In a typical communication system, if an input signal is given, then an output signal (*y)* should be produced by the system. In an ideal situation, if there is no noise input, the output *y* will be consistent and has no variation. However, with noise input, the output (*y)* will not be consistent and will vary. For a good communication system, the noise effect should have minimal influence in comparison with the output(y) therefore, the following signal-to-noise ratio (S/N) is often used as a quality characteristic of a communication system:[16]

$$\frac{Signal\ Power}{NoisePower} = \frac{\mu^2}{\sigma^2}$$

where $E(y) = \mu$ *and* $\operatorname{var}(y) = \sigma^2$

therefor the SN of Nominal –the best is

$$S/N = 10Log\left(\frac{\mu^2}{\sigma^2}\right) \qquad \text{... 1-30}$$

The use of a logarithm of a signal-to-noise ratio is based on the fact that the logarithm will transform a multiplicative relationship into an additive relationship; thus the logarithm transformation will “smooth out” lots of nonlinearities and interactions, which are desirable in data analysis. In real-world conditions, both μ^2 and $ó^2$ can only be estimated statistically. [39]

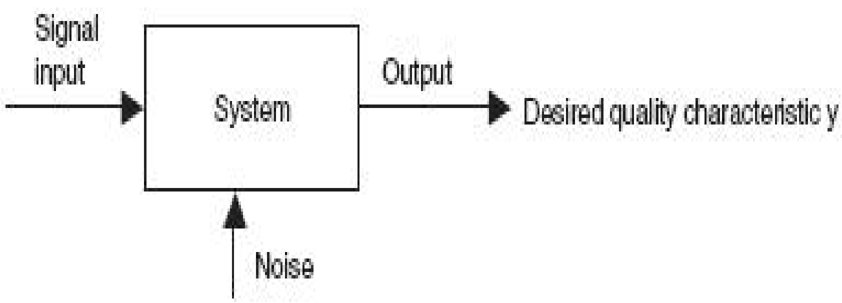


Figure (1-15) A typical communication system.

Therefore, the S/N ratio calculation for a set of real output performance data $y_1, y_2, \ldots, y_n$ is given by:

$$S/N = 10Log\left(\frac{\mu^2}{\sigma^2}\right) = 10Log\left(\frac{\bar{y}^2}{S^2}\right) = 20Log\left(\frac{\bar{y}}{S}\right) \qquad \text{... 1-31}$$

Comparing this with the nominal-the-best quality loss function, is obtained

$$L(y) = k(\mu - T)^2 + k\sigma^2 \qquad \text{... 1-32}$$

Taguchi proposed a two-step optimization procedure:

1. Adjust design parameters to maximize the S/N ratio.

2. Find some other design parameters that have no effect on S/N ratio but affect the mean level of Y, $E(Y)$, which is called the (*mean adjusting parameter)*, and then use it to tune $E(Y)$ to the target value.

One can see that by applying these two steps, $L(Y)$ will be minimized.

This two-step procedure was challenged and criticized by many people, including Box (1988), and alternative approaches have been proposed.

However, in communication engineering, maximizing S/N first and then adjusting the mean to the target is a common practice.

For many other non commutation engineering systems, the idea of minimizing the influence of “noise factors” and being robust to all sources of variation has also proved to be very valuable.

1-14-2 When there is No Signal Factor (Nondynamic SN ratio) [13][15][14][21]

One of the most important applications of dynamic SN ratio is to improve the robustness of product or process function within a certain output range.

In nondynamic SN ratio, there are two issues:

1- To reduce variability.

2- To adjust the average to the target which is reduced and adjusted to the target. There are two types :

I - Type one is the case where the data are all nonnegative.

II –Type two is the case where the data are a mixture of positive and negative values.

In two steps optimization for a nominal –the-best application, the first step is to maximize the SN ratio and thereby minimize variability around the average .The second step is to adjust the average to the target.

The nominal –the- best SN ratio is described as:

$$SN = \frac{desired\ output}{undesired\ output}$$

$$= \frac{effect\ \ of\ \ the\ average}{\text{var}iability\ around\ the\ average}$$

Calculations of the a nominal –the-best SN ratio if the data are not negative for (n) of data ;

1- Compute S/N average of Variation

$$S_n = \frac{(y_1 + y_2 + \cdots + y_n)^2}{n} \qquad \text{....1-33}$$

2- Compute Total Sum of squares

$$S_T = y_1^2 + y_2^2 + \cdots + y_2^2 \qquad \text{....1-34}$$

3- Compute Error Sum of Square or Variation

$$S_e = S_T - S_n \qquad \text{....1-35}$$

4- Compute the Error Variance

$$V_e = \frac{S_e}{n-1} \qquad \text{....1-36}$$

5- Compute SN ratio denoted by η and given by

$$\eta = 10 \log \left(\frac{1}{n} \frac{S_n - V_e}{V_e} \right) \qquad \text{....1-37}$$

6- And the Sensitivity denoted by S and given By :

$$S = 10\, Log\ \frac{1}{n}(S_m - V_e) \qquad \text{....1-38}$$

1-14-3 Smaller-the-better SN ratio quality characteristics

The smaller-the-better quality loss function is given by

$$L(y) = k(MSD)$$

$$MSD = \sum_{i=1}^{n} \frac{y_i^2}{n} = \sigma^2 + \bar{y}^2$$

$$L(y) = kE(Y^2)$$

In the real world, $E(Y^2)$ can only be estimated statistically. If a set of observations of quality characteristic Y are given, that is, $\boldsymbol{y_1, y_2, \ldots, y_n}$,

where MSD is Mean Squared Deviation from the target value .

The signal-to-noise ratio in this case is defined as

$$S/N = -10\,Log\left(\frac{1}{n}\sum_{i=1}^{n} y_i^2\right) \qquad \text{....1-39}$$

Clearly, S/N smaller – the-better obtains when the objects are to minimize the output value and there are no negative data. In such cases, the target value is zero, and, -10 times the logarithm of MSD; the smaller the MSD, the larger the S/N ratio. Therefore, for the smaller-the-better quality loss function, maximizing S/N is equivalent to minimizing the loss function.

1-14-4 S/N ratio Larger-the-better quality characteristics

The larger-the-better is obtained when the intention is to maximum the output and there are no negative data . In each case the target value is infinity.

The quality loss function is given by

$$L(y) = kE\left(\frac{1}{y^2}\right) \qquad \text{....1-40}$$

If a set of observations of quality characteristic Y are given, that is,

$\boldsymbol{Y_1, Y_2, \ldots, Yn}$, the statistical estimate of $E\left(\frac{1}{y^2}\right)$.

The calculation of the larger –the –batter SN ratio is given by:

$$S/N = -10\,Log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y^2}\right) \qquad \text{.... 1-41}$$

Again, maximizing S/N is equivalent to minimizing the quality loss function.

1-14-5 The Gain and Optimum of S/N Ratios [15][42]

The optimum of S/N ratio calculate as this equation :

Y_{Actual} S/N = T - (A_1-T) + (B1-T) + (C1-T) + (D1-T) + (E1-T) + (F1-T) + (J1-T) + (K-T) + (L1-T) + (M2-T)

$$=(\sum_{i}^{k} R_i) - kT \qquad \text{....1-42}$$

Where

k **= Number of variables**

R **= The value of S/N ratio of level**

T **= Total of SN**

And to define the gain by the following equation

Gain =(Average S/N Ratio)$_{\text{Level }1}$ – (Average S/N Ratio)$_{\text{Level }2}$ …1-43

$$Variability\ Reduction = 1-(0.5)^{Gain/6} \qquad \text{...1-44}$$

1-14-6 Robust parameter design using Signal-to-noise ratio and orthogonal array experiment[18]

In Taguchi’s robust parameter design approach, a number of design parameters will be selected and an orthogonal array for the experiment will selected. In each experimental run, several replicates of output performance observations will be collected as shown in table (1-1).

The signal-to-noise ratio is computed for each experimental run

Table(1-1)Taguchi Robust Design [16]

Taguchi Robust Parameter Design Setup

Experiment no.	Design parameters A	B	...	Replicates 1	...	n	S/N η
1	1	1	...	y_{11}	...	y_{1n}	η_1
2	2	1	...	y_{21}	...	y_{2n}	η_2
⋮	1	2	...	⋮		⋮	⋮
	2	2	...				
	⋮	⋮					
N	2	2	...	y_{N1}	...	y_{Nn}	η_N

If the quality characteristics *y* is either of smaller-the-better or larger the-better type, then the researchere will try to find the design parameter level combination to maximize S/N, thus minimizing the quality loss.

If the quality characteristic *y* is the nominal-the-best type characteristic, the following two steps will performed:-

1. Find and adjust significant design parameters to maximize S/N.

2. Find the mean adjustment design parameter to tune the mean response to the target value.

1-15-1 The role of orthogonal arrays [14][15][44]

In robust engineering the main role of OAs is to permit engineers to evaluate a product design with respect to robustness against noise and cost involved .The OA is an inspection device to prevent a "poor design" from going "downstream". Usually these arrays are denoted as $L_n(X^Y)$ arrays that have factors with many levels although two and three factors are the most commonly encountered .

Symbol $L_n(X^Y)$

Where

L = denotes Latin square design

n = Number of experiments

X = Number of levels

Y = Number of factors

The purpose of using the orthogonal arrays in the robust design or the design experiments is to estimate the effects of several factors and required interaction by minimizing the number of experiments.

In MTS, OA's are used to identify the useful variables with minimum number of variable combinations. The variables are assigned to the different columns of the array.

The presence and the absence of the variables considered as the different levels, are used , suppose that there are five variables with two levels, then $L_8(2^7)$ array are shown as in table .

Table (1-2) variable allocation in $L_8(2^7)$

	X1	X2	X3	X4	X5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

Therefor the first combination :1-1-1-1-1 ,

This means the combination is with X_1,X_2,X_3,X_4,X_5 the correlation matrix is of the order (5*5) ,The MD's of the abnormal are estimated with this matrix.

The Second Run 1-1-1-2-2

In this run MS is constructed with the variable combination X_1,X_2,X_3

The correlation matrix is of the order (3*3), the MDs of the abnormal are estimated with this matrix.

The Third run

In this run with variables combination is with X_1 ,X_4 ,X_5 ,and the matrix is of the order (3*3). For all combinations of the array, MD's corresponding to the abnormal condition are computed. Only abnormal are used , because it is required to ensure the accuracy of the measurement scale known abnormal.

1-15-2 Role of S/N ratio[14]

In multidimensional applications, it is important to identify a useful set of variables that is sufficient to detect the abnormal . It is also important to assess the performance of the given system and the degree of the improvement in the performance . In this method SN ratio is used to accomplish these objectives .

After obtaining the MD's for the known abnormal conditions corresponding to the various combinations of an OA, SN ratios are computed for all these combinations to determine the useful set of variables. Higher value of S/N ratio means lower error. S/N ratio are important to improve the accuracy of the measurement scale and reduce the cost of diagnosis.

The useful set of variables is obtained by evaluating the " gain" in S/N ratio to the gain in SN which is a Difference between the average S/N ratio when the variable is used in OA and the average S/N when the variable is not used in OA .

If the difference or the gain is positive , then the variable is useful. The reason for using orthogonal arrays is not to reduce cost by improving the efficiency of experimentation ,but to check the reproducibility of conclusions by conducting confirmatory experiments. The success or failure will be clear.

1-15-3 Standard Notations for Orthogonal Arrays [14][15][26]

For the elements of the orthogonal array (-, +), (0; 1) or (1; 2), as preferred by Taguchi, can be used. The array $L_4(2^3)$ which is the smallest of the 2-element orthogonal arrays, is given in Table (1-3)

Table (1-3). The Orthogonal Array $L_4(2^3)$.

	Column		
Row	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

As can be seen from Table (1-3), the 4 ordered pairs, (1; 1), (1; 2), (2; 1) and (2;2),appear exactly once in every pair of columns, so the orthogonal constraint is ensured.

This also means that elements 1 and 2 occur in the same number of times, that is twice, in each row Orthogonal arrays can be viewed as factorial experiments, where the columns correspond to factors and interactions, the entries in the columns correspond to the levels of the factors and the rows correspond to runs. An important feature of orthogonal arrays is that a sub matrix formed by deleting some columns and interchanging rows or columns is an orthogonal array too. The 2-level orthogonal arrays in Taguchi's catalogue, and the number of runs (rows),factors and interactions (columns) are given in Table(1-4)

Table (1-4). The 2-Level Orthogonal Arrays of Taguchi

Orthogonal arrays	Number of rows	Number of columns
$L_4(2^3)$	4	3
$L_8(2^7)$	8	7
$L_{12}(2^{11})$	12	11
$L_{16}(2^{15})$	16	15
$L_{32}(2^{31})$	32	31
$L_{64}(2^{63})$	64	63

Chapter Two

Practical Part
MTS and MTGS

Chapter Two

Section One

Application

2-1 The Data Analysis Of Water's Components :

Use of MTS and MTGS methods

This Chapter introduces the methodologies of the Mahalanobis Distance MD to compare MTS and MTGS methods, Basic steps in these methods are described with the help of chemical data analyses of water, the data for (Directorate Of Health Sulaimanyah-Directorate Of Prevention Health) are taken in different regions within the province of Sulaymaniyah.

All samples have (12) variables as shown in table(2-1) , each variable with two levels for two kinds of set data (260) samples are Normal and (80) sample are abnormal data.

Table(2-1) : Variables in Chemical data (Components of water analysis)

No.	Variables	Notation For Analysis
1	PH	X1
2	Electrical Conductivity (EC)	X2
3	Total Dissolved Solid (TDS)	X3
4	Sodium (Na)	X4
5	Potassium (K)	X5
6	Total hardness (T.H)	X6
7	Chloride (CI)	X7
8	Calcium (Ca)	X8
9	Calcium Hardiness	X9
10	Magnesium (Mg)	X10
11	Alkalinity	X11
12	Nitrate (Ni)	X12

There are two primary methods for computing the MD ,

1- Mahalanobois Taguchi System - MTS

2-Mahalanobois Taguchi Gram-Schmidt Methods - MTGS.

The Data for this application contains observations of a normal group as well as abnormal conditions.

I- A normal group that consists of (260) samples is acceptable in the chemical analysis, this data are called healthy data, as shown in table(2-2).

II- An abnormal group , these data are not in the range of healthy limit and it is called abnormal data, with samples of size(80), as shown in table(2-7)

Table(2-2) Normal Group Data

				Normal Data 260 sample								
Sample No	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
1	7.68	783	501	8.9	1	370	31.8	3.2	8	88	222	47
2	7.51	514	329	23.7	1	290	37.6	10	24	65	275	16
3	7.49	380	243	6.2	0.2	270	237	8	20	61	209	23
4	7.56	333	213	7.9	0.1	240	46.8	37	92	36	272	22
5	7	555	355	15.3	0.8	360	57	8	20	82	206	39
6	7.87	343	219	21.5	1.2	200	31	23	57	34	224	22
7	7.99	475	304	14.9	1.2	80	56	8	20	14	290	1
8	7.68	345	220	4.5	0.7	190	32	26	65	30	252	8
9	7.38	509	325	11.6	0.7	330	50	36	92	57	231	39
10	7.3	285	195	13.4	1.1	190	55	24	60	31	204	29
11	7.4	446	285	7.5	0.4	260	49	33	84	42	223	40
12	7.3	307	196	7.3	0.5	200	32	22	49	36	264	6
13	6.8	608	389	23.3	2.2	320	61	54	136	44	292	50
14	7.2	367	234	4	0.7	210	25	19	48	32	228	10
. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
250	7.4	464	296	5	1.3	340	54	25	64	67	269	4
251	7.49	424	271	26.6	1.1	220	41	25	64	37	211	2
252	7.2	569	364	6.3	0.9	320	63	22	56	64	280	19
253	7.9	433	277	5	0.6	300	6	28	72	55	205	12
254	7.04	521	333	1.9	0.8	280	10	19	48	56	297	23
255	7.28	535	342	4.8	0.5	330	15	16	40	70	271	3
256	7.09	873	558	31.5	0.6	480	22	22	56	103	293	37
257	6.91	546	349	20.2	1.4	340	20	41	104	57	218	14
258	6.9	715	457	35.7	0.9	340	17	12	32	74	284	15
259	7.47	405	634	45.3	1.4	400	11	51	128	66	235	21
260	8.03	277	434	19.2	1.6	300	12	32	80	53	225	13

Case One:

2-2 Mahalanobois Distance: MD

2-2-1 **MD-MTS for normal group** :

The basic steps to determine the MD -MTS are :

Step 1:

Define the Normal Group with (n=260) as shown in Table(2-2)

Step 2:

Define the variables X1, X2 ,X3X4,…,X12 as in table(2-1)

Step 3:

Compute the mean and the standard deviation for each variable and Standard the data, see Table(2-3) below.

Table(2-3) normalized **values for a Normal data group**

#	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10	Z11	Z12
1	1.737	1.933	1.885	-0.34	0.014	1.586	-0.67	-2.15	-2.18	2.324	-0.64	2.183
2	1.21	-0.09	-0.09	0.547	0.014	0.504	-0.5	-1.44	-1.52	1.022	1.318	-0.33
3	1.148	-1.09	-1.08	-0.5	-1.1	0.233	5.56	-1.65	-1.68	0.796	-1.12	0.24
4	1.365	-1.45	-1.43	-0.4	-1.24	-0.17	-0.22	1.352	1.306	-0.62	1.207	0.159
5	-0.37	0.222	0.204	0.043	-0.26	1.45	0.092	-1.65	-1.68	1.985	-1.23	1.535
6	2.326	-1.37	-1.36	0.415	0.292	-0.71	-0.7	-0.1	-0.15	-0.73	-0.57	0.159
7	2.697	-0.38	-0.38	0.019	0.292	-2.34	0.061	-1.65	-1.68	-1.87	1.873	-1.54
8	1.737	-1.36	-1.35	-0.61	-0.4	-0.85	-0.67	0.214	0.184	-0.96	0.467	-0.97
9	0.807	-0.12	-0.14	-0.18	-0.4	1.045	-0.12	1.249	1.306	0.569	-0.31	1.535
10	0.559	-1.81	-1.64	-0.07	0.153	-0.85	0.031	0.007	-0.02	-0.9	-1.31	0.725
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
251	1.148	-0.76	-0.76	0.721	0.153	-0.44	-0.39	0.111	0.143	-0.56	-1.05	-1.46
252	0.249	0.327	0.308	-0.5	-0.12	0.909	0.274	-0.2	-0.19	0.965	1.503	-0.08
253	2.419	-0.69	-0.69	-0.58	-0.54	0.639	-1.46	0.421	0.475	0.456	-1.27	-0.65
254	-0.25	-0.03	-0.05	-0.76	-0.26	0.368	-1.34	-0.51	-0.52	0.513	2.132	0.24
255	0.497	0.071	0.055	-0.59	-0.68	1.045	-1.18	-0.82	-0.85	1.305	1.17	-1.38
256	-0.09	2.609	2.54	1.014	-0.54	3.073	-0.97	-0.2	-0.19	3.174	1.984	1.373
257	-0.65	0.154	0.135	0.337	0.57	1.18	-1.03	1.766	1.804	0.569	-0.79	-0.49
258	-0.68	1.423	1.378	1.266	-0.12	1.18	-1.12	-1.23	-1.19	1.532	1.651	-0.41
259	1.086	-0.9	3.415	1.842	0.57	1.991	-1.31	2.801	2.801	1.079	-0.16	0.078
260	2.821	-1.87	1.114	0.277	0.848	0.639	-1.28	0.835	0.807	0.343	-0.53	-0.57

Step 4:

Construct the Correlation Coefficients matrix of normalized data as shown in Table(2-4).

Table(2-4) Correlation Coefficients of Normal Data

	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10	Z11	Z12
Z1	1	-0.36	-0.28	0.032	-0.09	-0.11	-0.19	-0.21	-0.22	-0.02	-0.1	-0.29
Z2	-0.36	1	0.909	0.305	0.184	0.305	0.373	0.238	0.237	0.255	0.326	0.468
Z3	-0.28	0.909	1	0.306	0.182	0.351	0.336	0.259	0.255	0.295	0.327	0.432
Z4	0.032	0.305	0.306	1	0.286	-0.07	0.275	0.068	0.056	-0.03	0.102	0.108
Z5	-0.09	0.184	0.182	0.286	1	0.125	0.082	0.202	0.193	0.071	0.136	0.077
Z6	-0.11	0.305	0.351	-0.07	0.125	1	0.09	0.285	0.289	0.91	0.186	0.194
Z7	-0.19	0.373	0.336	0.275	0.082	0.09	1	0.153	0.152	0.051	0.096	0.292
Z8	-0.21	0.238	0.259	0.068	0.202	0.285	0.153	1	0.981	-0.02	0.134	0.191
Z9	-0.22	0.237	0.255	0.056	0.193	0.289	0.152	0.981	1	-0.02	0.141	0.17
Z10	-0.02	0.255	0.295	-0.03	0.071	0.91	0.051	-0.02	-0.02	1	0.143	0.154
Z11	-0.1	0.326	0.327	0.102	0.136	0.186	0.096	0.134	0.141	0.143	1	-0.03
Z12	-0.29	0.468	0.432	0.108	0.077	0.194	0.292	0.191	0.17	0.154	-0.03	1

Step 5:

The inverse matrix of the correlation matrix is constructed as shown in Table(2-5)

Table(2- 5) Inverse matrix of Correlation Coefficients of Normal Data

	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10	Z11	Z12
Z1	1.276	0.712	-0.36	-0.2	0.063	0.195	0.072	-0.28	0.369	-0.26	0.011	0.182
Z2	0.712	6.628	-5.38	-0.2	-0.06	0.33	-0.32	0.534	-0.46	-0.25	-0.32	-0.51
Z3	-0.36	-5.38	6.161	-0.26	0.043	-0.56	0.061	-0.38	0.197	0.082	-0.17	-0.11
Z4	-0.2	-0.2	-0.26	1.39	-0.34	1.126	-0.27	-0.28	0.024	-0.84	0.006	0.061
Z5	0.063	-0.06	0.043	-0.34	1.158	-0.27	0.057	-0.29	0.178	0.165	-0.08	0.008
Z6	0.195	0.33	-0.56	1.126	-0.27	13.79	-0.16	-0.17	-3.92	-12.5	-0.17	0.026
Z7	0.072	-0.32	0.061	-0.27	0.057	-0.16	1.247	0.096	-0.12	0.168	-0	-0.19
Z8	-0.28	0.534	-0.38	-0.28	-0.29	-0.17	0.096	26.86	-26.2	0.174	0.072	-0.82
Z9	0.369	-0.46	0.197	0.024	0.178	-3.92	-0.12	-26.2	27.91	3.65	-0.1	0.719
Z10	-0.26	-0.25	0.082	-0.84	0.165	-12.5	0.168	0.174	3.65	12.4	0.073	-0.11
Z11	0.011	-0.32	-0.17	0.006	-0.08	-0.17	-0	0.072	-0.1	0.073	1.203	0.289
Z12	0.182	-0.51	-0.11	0.061	0.008	0.026	-0.19	-0.82	0.719	-0.11	0.289	1.439

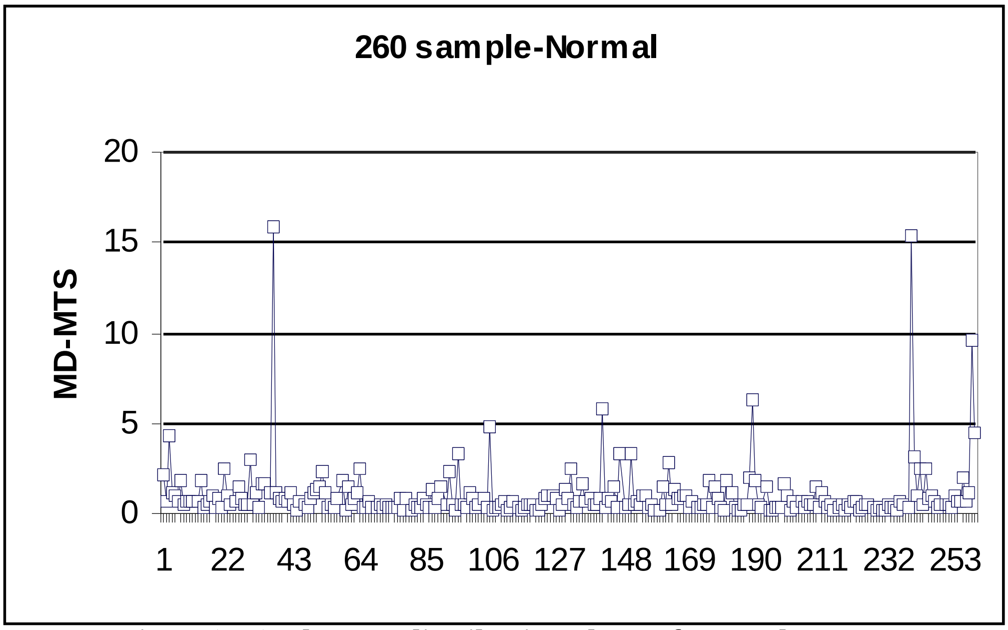
Step 6:

By using inverse correlation matrix and normalized data, Mahalanobois distance MD are calculated as shown in Table(2-6)

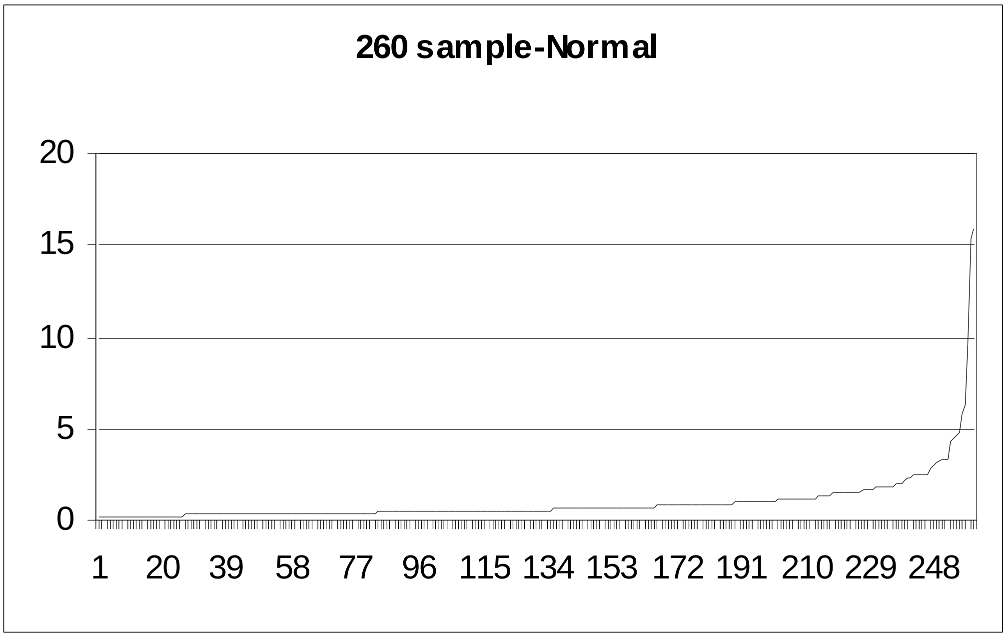
Table(2-6) MD-MTS Mahalanobois distances of 260 samples for normal data .

No.	MD	No.	MD	No.	MD	No.	MD	No.	MD	No.	MD
1	2.195	46	0.545	91	0.445	136	0.717	181	1.851	226	0.525
2	0.589	47	0.34	92	2.25	137	0.766	182	1.181	227	0.254
3	4.275	48	0.886	93	0.517	138	0.525	183	0.374	228	0.177
4	1.141	49	1.159	94	0.102	139	0.52	184	0.197	229	0.372
5	0.944	50	1.252	95	3.235	140	0.781	185	0.133	230	0.153
6	0.735	51	1.453	96	0.479	141	5.734	186	0.437	231	0.222
7	1.755	52	2.24	97	0.435	142	0.381	187	0.483	232	0.494
8	0.507	53	1.108	98	0.406	143	0.755	188	1.931	233	0.265
9	0.671	54	0.358	99	1.219	144	0.714	189	6.361	234	0.401
10	0.628	55	0.455	100	0.831	145	1.466	190	1.851	235	0.208
11	0.714	56	0.398	101	0.261	146	0.303	191	0.489	236	0.676
12	0.667	57	0.893	102	0.459	147	3.243	192	0.262	237	0.551
13	1.785	58	1.746	103	0.822	148	0.364	193	1.489	238	0.319
14	0.314	59	0.215	104	0.388	149	0.448	194	0.207	239	15.43
15	0.575	60	1.498	105	4.831	150	3.368	195	0.119	240	3.097
16	0.61	61	0.541	106	0.191	151	0.524	196	0.305	241	0.921
17	1.028	62	0.867	107	0.364	152	0.598	197	0.393	242	2.5
18	0.357	63	1.086	108	0.409	153	0.546	198	0.26	243	0.562
19	0.874	64	2.448	109	0.577	154	1.011	199	1.64	244	2.526
20	0.257	65	0.351	110	0.734	155	0.911	200	1.022	245	0.894
21	2.554	66	0.332	111	0.2	156	0.353	201	0.239	246	0.918
22	1.068	67	0.609	112	0.276	157	0.461	202	0.587	247	0.609
23	0.42	68	0.307	113	0.694	158	0.169	203	0.405	248	0.409
24	0.667	69	0.313	114	0.336	159	0.236	204	0.712	249	0.434
25	1.474	70	0.425	115	0.146	160	1.494	205	0.254	250	0.495
26	0.856	71	0.294	116	0.408	161	0.516	206	0.714	251	0.452
27	0.423	72	0.416	117	0.529	162	2.878	207	0.472	252	0.309
28	0.429	73	0.373	118	0.566	163	0.458	208	0.49	253	0.95
29	2.993	74	0.324	119	0.522	164	1.346	209	1.518	254	0.74
30	0.511	75	0.278	120	0.115	165	0.748	210	0.25	255	0.621
31	1.202	76	0.305	121	0.201	166	0.748	211	1.119	256	1.936
32	0.37	77	0.836	122	0.476	167	0.983	212	0.601	257	0.733
33	1.69	78	0.123	123	0.857	168	0.998	213	0.271	258	1.154
34	1.646	79	0.861	124	0.933	169	0.32	214	0.425	259	9.528
35	1.183	80	0.203	125	0.941	170	0.736	215	0.165	260	4.432
36	15.88	81	0.459	126	0.811	171	0.331	216	0.261		
37	1.11	82	0.391	127	0.214	172	0.342	217	0.514		
38	0.75	83	0.384	128	0.417	173	0.417	218	0.222		
39	0.705	84	0.772	129	1.295	174	0.484	219	0.473		
40	0.819	85	0.534	130	0.852	175	1.836	220	0.363		
41	0.683	86	0.334	131	2.534	176	0.287	221	0.73		
42	1.173	87	1.354	132	0.439	177	1.486	222	0.596		
43	0.47	88	0.344	133	0.339	178	0.882	223	0.198		
44	0.185	89	0.795	134	0.587	179	0.405	224	0.386		
45	0.728	90	1.497	135	1.599	180	0.195	225	0.443		

Fig.(2-1) shows the distribution chart of MDs value for 260 observation of 12 variables calculated by MTS methods, and Fig.(2-2) is a sorting chart of MD value, it is seen that (96.9 %) of the MD value is less than (4).



Figure(2-1) The MD distribution chart of normal group



Figure(2-2) The Sort Chart of MTS For normal group

Table(2-9) contains some of the basic statistical analyses of MTS , it was found that the average of MD is (0.996154) this value is close to one and considered that this option is an important properties of MD , and the standard deviation value of MTS equals (1.65008414), the range of MD is between (0.1017867 and 15.880164).

From Fig.(2-3),(2-4) and from sorting the MD value, it shows that the largest eight value in 260 MTS are (4.28, 4.43, 4.84, 5.73, 6.36, 9.53, 15.4, 15.9), so it is possible to say that those eight points are considered as abnormal points compared with the other .

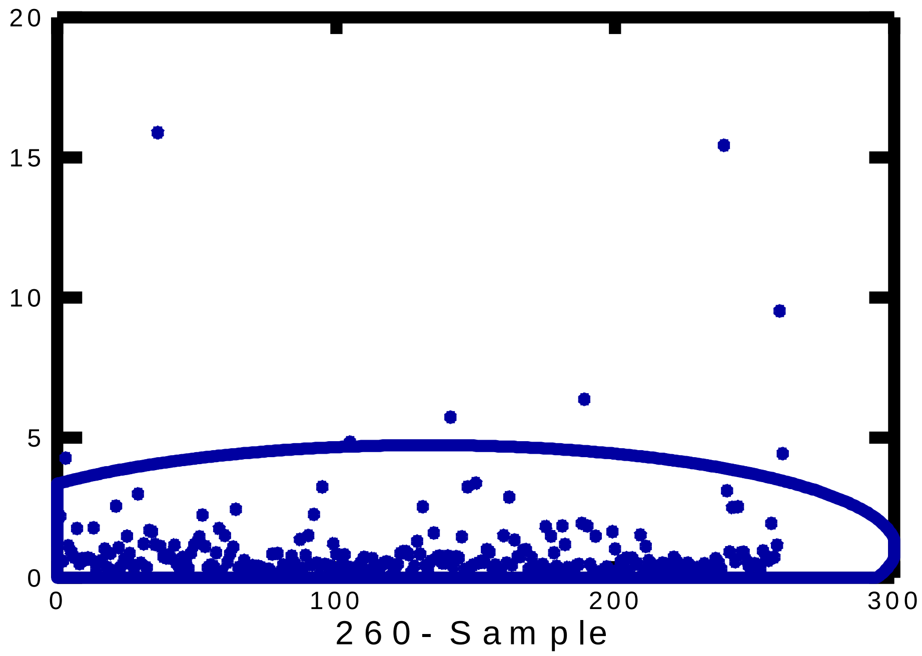
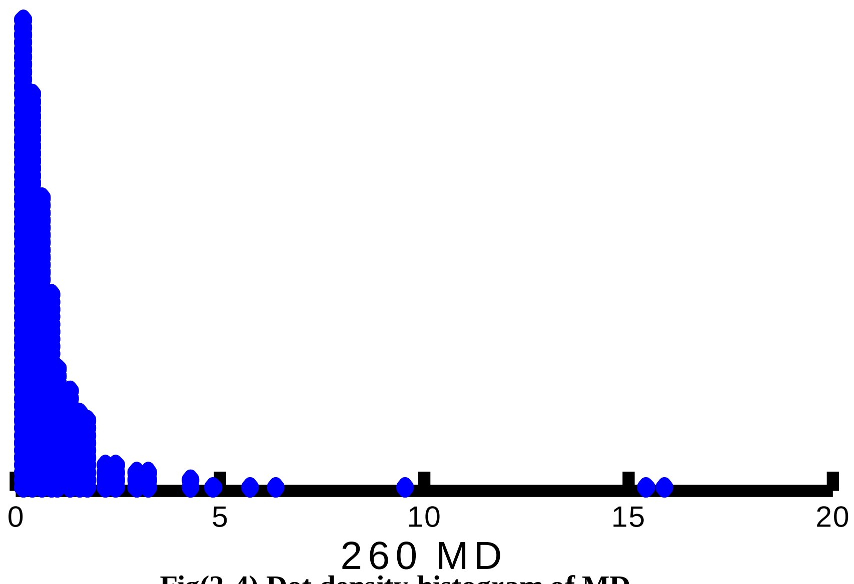


Fig.(2-3) Scatterplote Chart of Normal MD



Fig(2-4) Dot density histogram of MD

2-2-2 MD-MTS –for Abnormal group data with sample size (80)

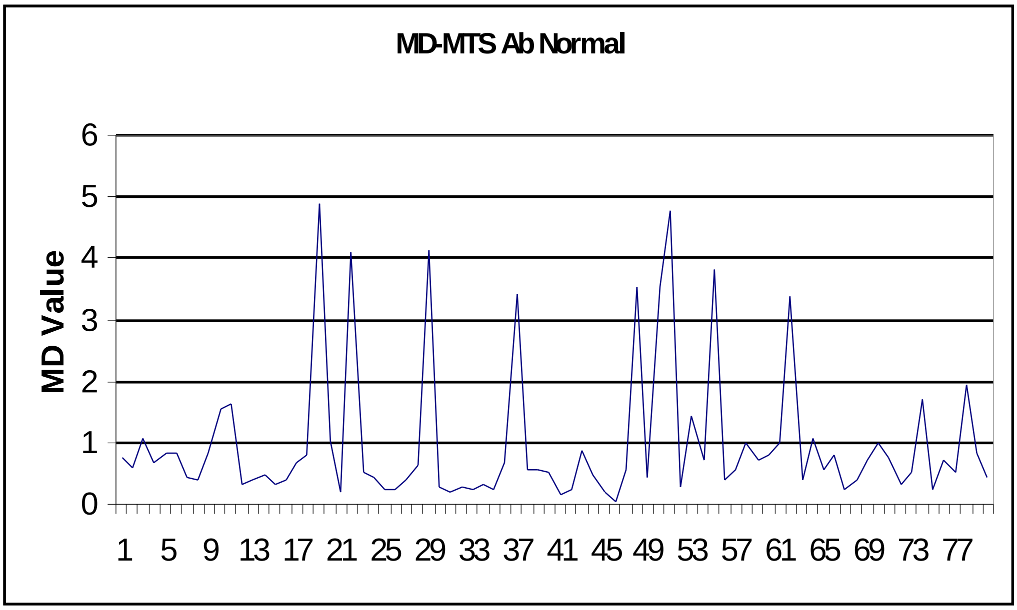
Compute the Values MD-MTS for the abnormal data, using the data in table(2-7) and Eq.(1-18), the values of MD-MTS are shown in table(2-8) and represented in Fig.(2-5).

Table(2-7) Abnormal data group sample size(80)

	Abnormal Data											
Sample No	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
1	7.58	1070	685	95.3	1.4	390	312	6.4	16	91	163	46
2	6.78	869	200	76.6	3.3	550	142	8	20	129	170	25
3	8.9	480	307	125	1.3	150	96	40	100	12	172	9
4	8.71	380	243	150	4.6	100	21.3	12	32	16	253	10
5	7.3	1614	1032	263	5.4	220	390	35	88	32	262	46
6	7.4	445	284	154	14.7	310	56	33	84	55	199	38
7	6.09	1016	650	131	2.4	360	295	40	101	62	112	39
8	7.5	869	556	28.2	3.6	400	118	55	145	63	394	35
9	6.8	1643	1045	231	3.1	240	362	49	122	29	307	46
10	6.8	1212	775	38.4	2	310	64	112	280	7	292	47
. . .	. . .	. . .	. . .							. . .	. . .	. . .
76	8.18	1174	751	216	2.9	90	103	12	32	14	378	8
77	6.4	742	474	12.3	12.1	320	144	54	136	44	215	46
78	8.77	873	558	176	4.1	40	285	16	40	59	180	32
79	7.4	1059	677	200	2	80	193	6.4	16	15	378	2
80	6.5	1136	727	15.7	0.4	480	125	41	104	91	355	42

Table(2-8) MD values using MTS for Abnormal group Data

#	MD-MTS	#	MD-MTS	#	MD-MTS	#	MD-MTS
1	0.755774	21	0.209005	41	0.174355	61	1.00971
2	0.578114	22	4.083597	42	0.241882	62	3.371063
3	1.055507	23	0.52709	43	0.854397	63	0.408974
4	0.657546	24	0.435541	44	0.458073	64	1.089796
5	0.840419	25	0.252506	45	0.207132	65	0.553193
6	0.832146	26	0.239094	46	0.044004	66	0.789377
7	0.435837	27	0.385983	47	0.541679	67	0.247384
8	0.381078	28	0.636775	48	3.517925	68	0.390383
9	0.84042	29	4.132184	49	0.446753	69	0.730948
10	1.547759	30	0.263297	50	3.534968	70	1.000748
11	1.637609	31	0.203501	51	4.769425	71	0.756079
12	0.319783	32	0.277366	52	0.280845	72	0.326165
13	0.402888	33	0.245246	53	1.432807	73	0.509203
14	0.463594	34	0.311592	54	0.704538	74	1.695856
15	0.305174	35	0.227692	55	3.814847	75	0.237454
16	0.392112	36	0.676265	56	0.407406	76	0.724275
17	0.68999	37	3.435224	57	0.539044	77	0.510722
18	0.806718	38	0.554658	58	0.981306	78	1.952429
19	4.869954	39	0.550989	59	0.699603	79	0.848454
20	1.025889	40	0.502882	60	0.787547	80	0.420453



Figure(2-5) MD- MTS for Abnormal group data

From Fig.(2-5) above it is shown that the pick of frequency is not homogenized . Fig.(2-6) shows the sorting chart of MD value . The basic statistics of MD value of Normal and abnormal shows in table (2-9) it seen that the range of MD value of abnormal is between(.044004 - 4.869954) the average is (0.987), and the stranded deviation is (1.1345) .The range between normal and Abnormal MD it seen that the rang in Normal MD is larger than range in abnormal MD, where the value of range in normal is (15.77838), but in abnormal is (4.825950) ,the range without eight abnormal points becomes (3.266108), and the range of abnormal data without four points is (3.770843). From Fig(2-6) it seen that most of MD values less than(3) and (88.7 %) of MD value less than (3). Fig(2-7)shows that the density dote chart of MD of abnormal value .

Table(2-9) Basic Statistic of MD values for abnormal group data

	Abnormal Data	Normal Data
N of Cases	**80**	**260**
Minimum	**0.044004**	**0.10178667**
Maximum	**4.869954**	**1.59E+01**
Range	**4.82595**	**1.58E+01**
Sum	**78.999994**	**2.59E+02**
Arithmetic Mean	**0.9874999**	**0.99615386**
Standard Error of Arithmetic Mean	**0.126861**	**0.10233387**
Standard Deviation	**1.1346792**	**1.65008414**
Variance	**1.287497**	**2.72277768**

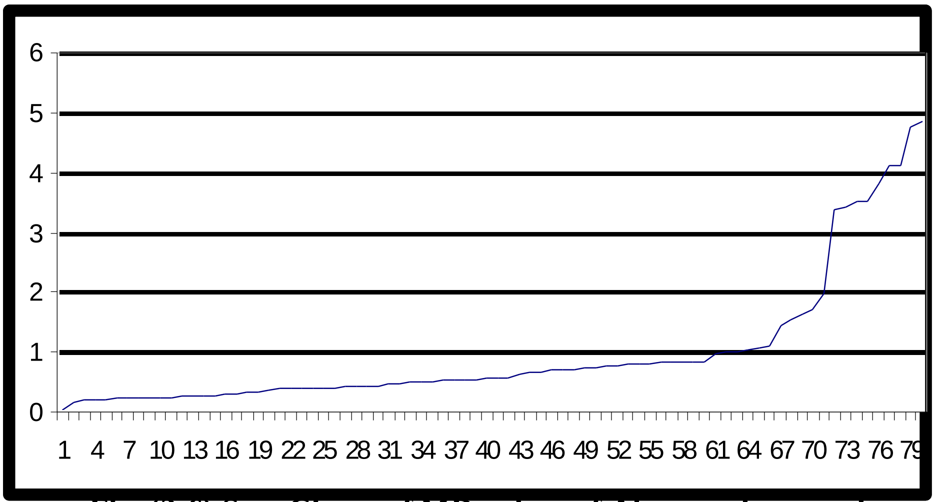
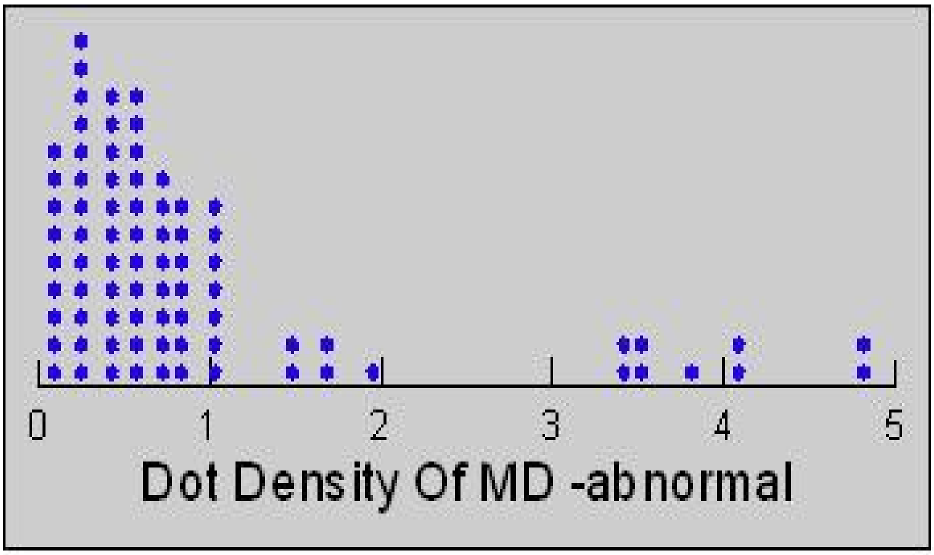
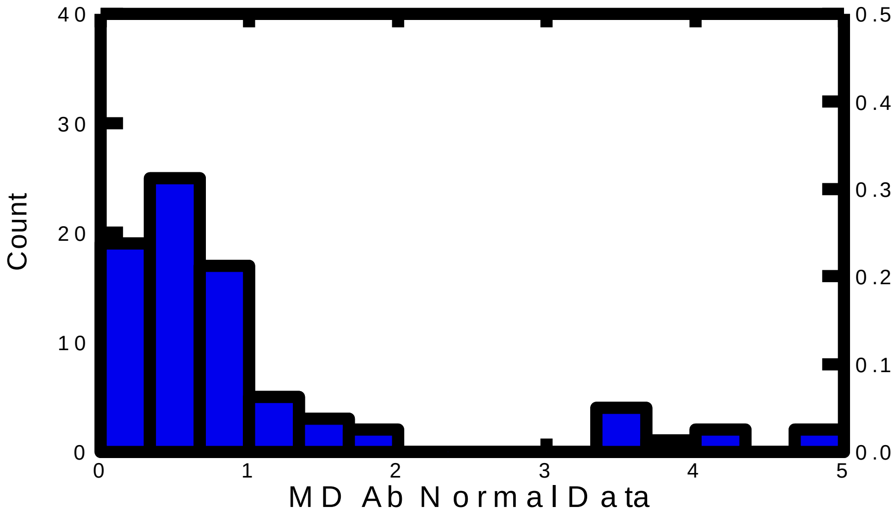


Fig. (2-6) Sort Chart of MD values of Abnormal group data

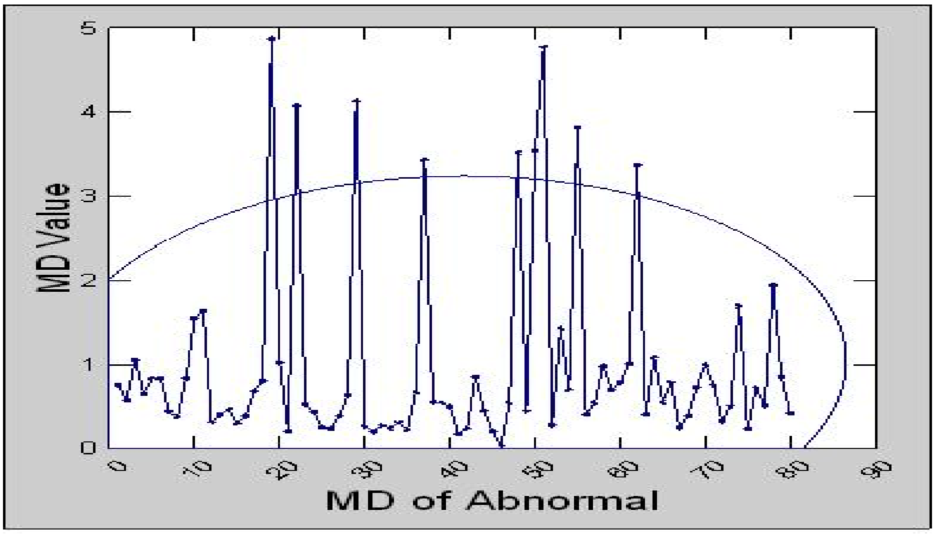


Figure(2-7) Dot Density Of MD Abnormal

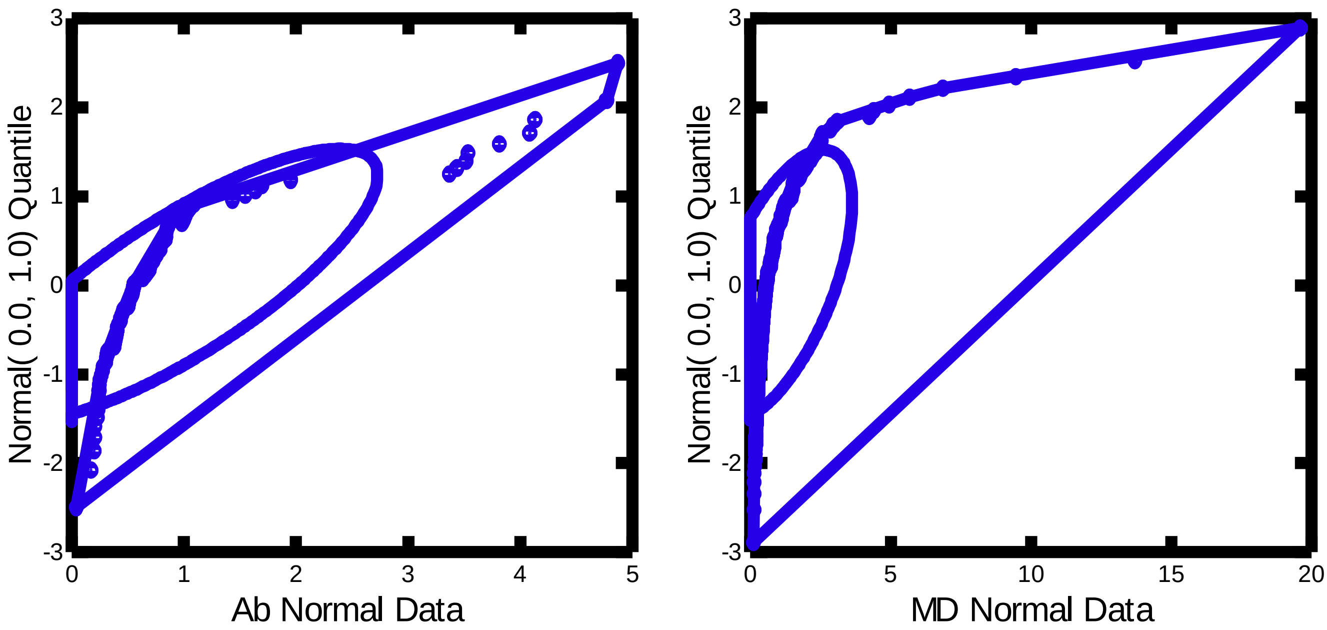
Fig(2-8) shows that the histogram distribution of MD value for mormal data , It seen that the most of MD value is less than two.



Figure(2-8) Histogram of MD Abnormal value



Figure(2-9) Scatterplote Chart of Abnormal value



Figure(2-10) Convex Hull For MD Of Abnormal And Normal and Confidence Ellipse

From above Fig.(2-10) it is seen that there is a difference in curve between abnormal and normal MD, it is clear that the convex curve for the normal data is better than the abnormal MD, and there are some points out of the confidence Ellipse.

Case Two:

2-3 Mahalanobis -Taguchi Gramm-Schmidt –Methods -MTGS

In this case calculate the MD value of Normal data with sample size(260) by Mahalanobis –Tagushi Gramm-Schmidt –Methods -MTGS .

MD-MTGS methods Steps :

Step-1.

Define the normal group with (sample size n=260), data as shows in table(2-1)

Step 2:

Define the variables (K=12), the variables define as table (2-1)

The Variables are :

X1, X2 ,X3, X4,…,X12

Step 3:

Compute the mean and standard deviations for each variable and normalize the all observation as this equation :

$$Z_{ik} = \frac{X_i - \bar{X}}{\sigma_x}$$

The result of normalizing data for sample size(260) shows in table(2-2)

Step 4:

Compute MD' s of all observations by using the Gram-Schmidt process as :

$$\boxed{Z_1, Z_2, Z_3, \ldots\ldots Z_k} \longrightarrow \boxed{\text{GSP}} \longrightarrow \boxed{U_1, U_2, U_3, \ldots U_k}$$

Computing the MD-MTGS by using the equations (1-20), (1-21) , Table (2-10) shows the value of vectors of ($U_1, U_2, \ldots. U_k$), .

($U_1, U_2, \ldots. U_k$), vectors and MD-MTGS was computed by using the special program on (Microsoft Office -Excel 2003) crated by researcher.

Table(2-10) The values of (U_1 ,U_2 ,…U_{12}) Gram-Schmidt vectors of 260 normal data

No.	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12
1	1.74	2.56	-0.02	-1.33	0.07	0.73	-1	-2.34	-0.08	0.01	-1.32	1.5
2	1.21	0.35	-0.09	0.39	-0.03	0.65	-0.48	-1.33	-0.09	-0.02	1.34	0.21
3	1.15	-0.68	-0.14	-0.28	-0.79	0.66	6.03	-1.67	-0.08	0.12	-0.5	-0.08
4	1.36	-0.95	-0.17	-0.08	-0.91	0.43	0.34	1.9	-0.01	-0.04	1.68	1.12
5	-0.37	0.09	0.02	0.02	-0.32	1.42	-0.05	-1.99	-0.09	0.01	-1.3	1.11
6	2.33	-0.52	-0.22	0.56	0.47	-0.14	-0.17	0.51	-0.01	-0.12	-0.17	1.06
7	2.7	0.6	-0.19	-0.26	0.52	-2.25	0.46	-0.78	-0.02	-0.08	2.14	-0.47
8	1.74	-0.72	-0.19	-0.37	-0	-0.43	-0.01	0.86	-0	-0.04	0.92	0.01
9	0.81	0.17	-0.07	-0.26	-0.29	1.1	-0.01	1.23	0.08	-0.09	-0.4	1.55
10	0.56	-1.6	0.01	0.49	0.35	-0.24	0.62	0.36	-0.02	-0.03	-0.72	1.33
11	0.87	-0.28	-0.1	-0.34	-0.59	0.33	0.15	1.22	0.06	-0.07	-0.44	1.83
12	0.56	-1.44	-0.13	0.09	-0.39	-0.09	-0.09	0.31	-0.28	-0.02	1.56	-0.21
13	-0.99	0.26	0.08	0.45	1.42	0.61	-0.11	2.47	0.08	-0.11	1.48	2.26
14	0.25	-1.1	-0.1	-0.23	-0.12	-0.21	-0.4	-0.13	-0.02	-0.39	-0.02	-0.19
15	-0.34	-0.91	-0.04	-0.22	1.73	0.4	-0.5	0.3	0.04	0.02	0.83	0.84
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
250	0.87	-0.15	-0.1	-0.54	0.68	1.17	0.32	-0.03	0.03	0.03	1.1	-0.85
251	1.15	-0.34	-0.12	0.82	0.11	-0.01	-0.21	0.44	0.06	-0.14	-0.85	-1.07
252	0.25	0.42	-0.01	-0.66	-0	0.68	0.29	-0.38	-0	0.02	1.36	0.02
253	2.42	0.18	-0.19	-0.7	-0.16	0.81	-0.96	0.88	0.09	-0.06	-1.19	-0.17
254	-0.25	-0.12	-0	-0.71	-0.08	0.25	-1.21	-0.49	-0.03	0.02	2.12	0.79
255	0.5	0.25	-0.04	-0.69	-0.49	0.96	-1.08	-0.79	-0.06	-0	1.12	-1.03
256	-0.09	2.58	0.12	0.06	-1.03	2.36	-1.98	-0.93	-0.01	-0.1	0.97	0.65
257	-0.65	-0.08	0.03	0.38	0.42	1.15	-1.2	1.4	0.07	-0.08	-1.05	-0.77
258	-0.68	1.18	0.1	0.85	-0.62	0.95	-1.87	-1.6	0.02	-0.13	1.14	-0.56
259	1.09	-0.51	4.19	1.49	-0.01	0.77	-1.22	1.81	0.11	-0.17	-0.77	0.26
260	2.82	-0.84	2.68	0.17	0.96	-0.02	-0.45	0.68	0.05	-0.08	-0.55	0.35
Average	0	0	0	0	0	0	0	0	0	0	0	0

From above table it seen that thae average of (U_1,U_2,….U_k), vectors are equal zero.

Table(2-11) contains the two cases of MD (MTS and MTGS) methods , it is seen that the values of MD in these cases are equal .

Table(2-11) MD values by (MTS and MTGS) methods for the Normal data

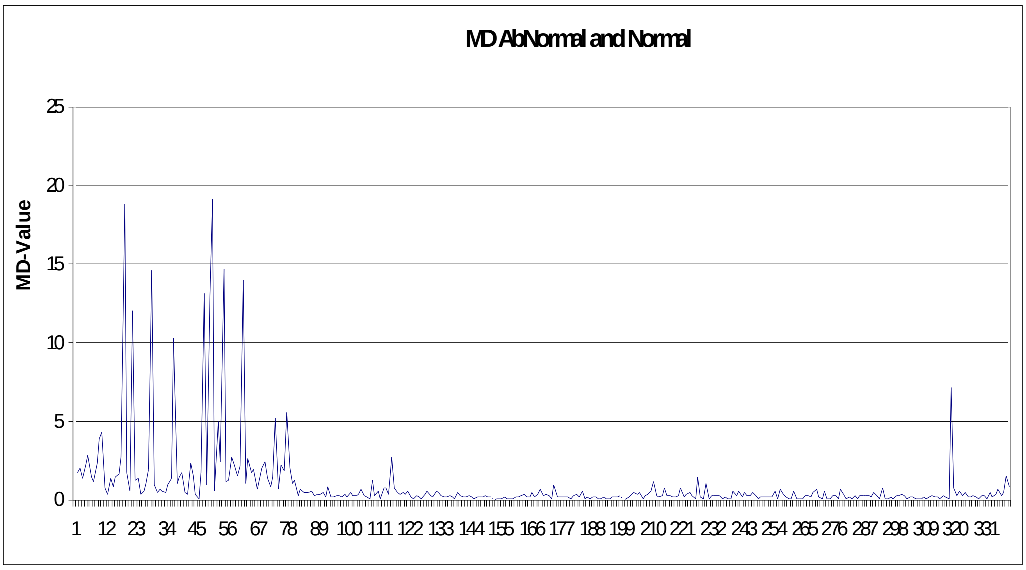
No.	MD-MTS	MD-MTGS
1	2.195199	2.195199
2	0.588689	0.588689
3	4.275480	4.275480
4	1.141436	1.141436
5	0.943675	0.943675
6	0.734665	0.734665
7	1.754866	1.754866
8	0.506894	0.506894
9	0.670796	0.670796
10	0.627885	0.627885
11	0.713828	0.713828
12	0.666801	0.666801
13	1.785003	1.785003
14	0.314303	0.314303
15	0.574639	0.574639
	. . . .	. . . .
250	0.495162	0.495162
251	0.451937	0.451937
252	0.308817	0.308817
253	0.949503	0.949503
254	0.740061	0.740061
255	0.621437	0.621437
256	1.936033	1.936033
257	0.732561	0.732561
258	1.153706	1.153706
259	9.528288	9.528288
260	4.432343	4.432343

2-4 Mixing the normal data group and the abnormal data group

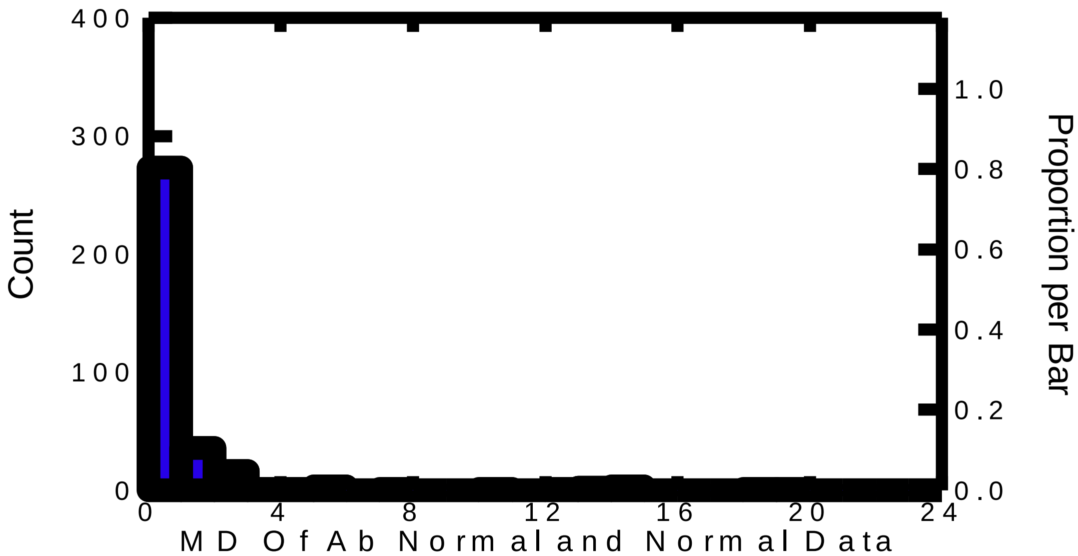
In this section calculate the MD value is calculated for mixing date of (Abnormal and Normal), the sample becomes (340), then The MD values are calculated as shown in Table(2-12) and Fig(2-11) .

Table(2-12) MD For Abnormal and Normal Data

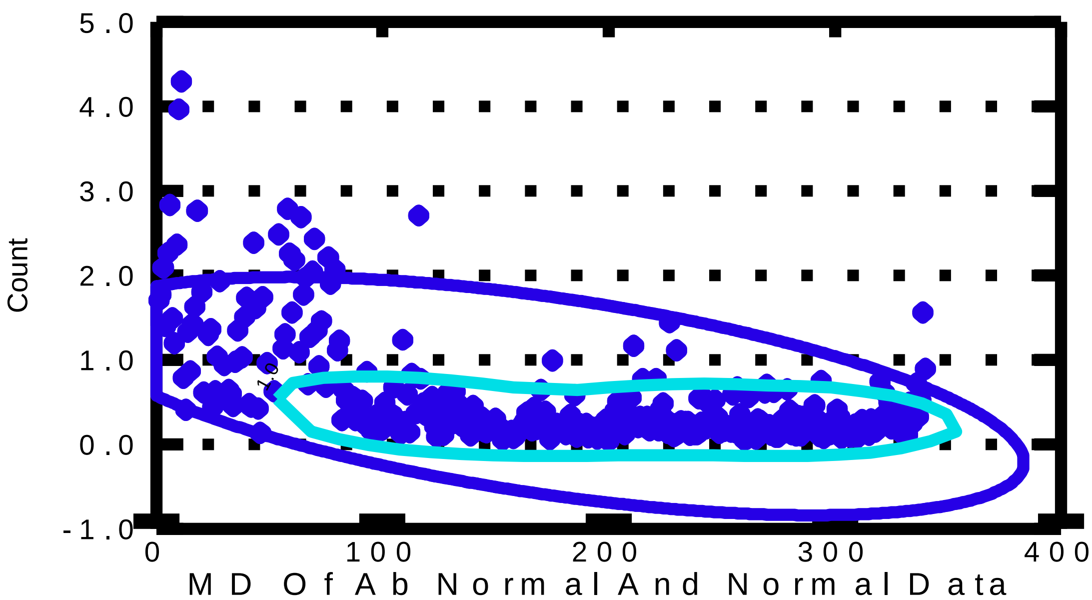
No	MD	No.	MD	No.	MD	No.	MD	No.	MD	No.	MD	No.	MD	No.	MD
1	1.701	46	0.136	91	0.515	136	0.22	181	0.116	226	0.133	271	0.227	316	0.296
2	1.774	47	1.738	92	0.243	137	0.334	182	0.254	227	1.442	272	0.114	317	0.236
3	2.09	48	13.15	93	0.858	138	0.236	183	0.345	228	0.239	273	0.616	318	0.146
4	1.396	49	0.959	94	0.207	139	0.106	184	0.191	229	0.092	274	0.084	319	7.137
5	2.262	50	13.07	95	0.156	140	0.452	185	0.582	230	1.11	275	0.088	320	0.739
6	2.834	51	19.15	96	0.274	141	0.265	186	0.084	231	0.142	276	0.254	321	0.331
7	1.493	52	0.63	97	0.336	142	0.2	187	0.242	232	0.275	277	0.292	322	0.612
8	1.195	53	5.046	98	0.162	143	0.236	188	0.146	233	0.247	278	0.14	323	0.27
9	2.36	54	2.483	99	0.348	144	0.314	189	0.191	234	0.263	279	0.652	324	0.499
10	3.966	55	14.75	100	0.172	145	0.15	190	0.237	235	0.262	280	0.402	325	0.189
11	4.298	56	1.133	101	0.492	146	0.143	191	0.081	236	0.11	281	0.118	326	0.211
12	0.783	57	1.304	102	0.259	147	0.176	192	0.103	237	0.226	282	0.183	327	0.336
13	0.405	58	2.787	103	0.34	148	0.176	193	0.193	238	0.116	283	0.1	328	0.209
14	1.333	59	2.256	104	0.358	149	0.181	194	0.101	239	0.115	284	0.328	329	0.141
15	0.866	60	1.559	105	0.669	150	0.303	195	0.066	240	0.544	285	0.105	330	0.276
16	1.425	61	2.178	106	0.303	151	0.161	196	0.216	241	0.255	286	0.327	331	0.295
17	1.629	62	14.04	107	0.194	152	0.199	197	0.161	242	0.552	287	0.257	332	0.113
18	2.763	63	1.092	108	0.127	153	0.049	198	0.244	243	0.186	288	0.268	333	0.488
19	18.85	64	2.689	109	1.236	154	0.123	199	0.295	244	0.488	289	0.316	334	0.236
20	1.8	65	1.769	110	0.262	155	0.129	200	0.038	245	0.303	290	0.211	335	0.366
21	0.612	66	1.973	111	0.59	156	0.13	201	0.11	246	0.321	291	0.458	336	0.716
22	12.02	67	0.707	112	0.144	157	0.158	202	0.208	247	0.524	292	0.299	337	0.325
23	1.297	68	1.272	113	0.829	158	0.07	203	0.398	248	0.324	293	0.099	338	0.512
24	1.365	69	2.041	114	0.795	159	0.1	204	0.505	249	0.137	294	0.748	339	1.56
25	0.438	70	2.429	115	0.37	160	0.142	205	0.401	250	0.204	295	0.076	340	0.889
26	0.624	71	1.345	116	2.708	161	0.205	206	0.446	251	0.154	296	0.115		
27	1.04	72	0.925	117	0.774	162	0.242	207	0.118	252	0.216	297	0.24		
28	1.929	73	1.457	118	0.48	163	0.279	208	0.247	253	0.163	298	0.117		
29	14.56	74	5.178	119	0.347	164	0.376	209	0.362	254	0.148	299	0.264		
30	0.94	75	0.682	120	0.494	165	0.243	210	0.553	255	0.592	300	0.304		
31	0.526	76	2.209	121	0.387	166	0.164	211	1.164	256	0.138	301	0.412		
32	0.645	77	1.903	122	0.557	167	0.432	212	0.27	257	0.675	302	0.32		
33	0.596	78	5.62	123	0.209	168	0.175	213	0.204	258	0.358	303	0.074		
34	0.453	79	2.066	124	0.085	169	0.368	214	0.325	259	0.197	304	0.24		
35	0.975	80	1.108	125	0.289	170	0.644	215	0.77	260	0.054	305	0.154		
36	1.347	81	1.228	126	0.219	171	0.275	216	0.285	261	0.069	306	0.141		
37	10.29	82	0.286	127	0.112	172	0.384	217	0.321	262	0.559	307	0.09		
38	1.031	83	0.644	128	0.415	173	0.276	218	0.164	263	0.14	308	0.141		
39	1.504	84	0.52	129	0.612	174	0.058	219	0.236	264	0.094	309	0.236		
40	1.733	85	0.526	130	0.282	175	0.988	220	0.255	265	0.064	310	0.081		
41	0.478	86	0.448	131	0.244	176	0.217	221	0.775	266	0.294	311	0.148		
42	0.44	87	0.563	132	0.623	177	0.239	222	0.163	267	0.281	312	0.286		
43	2.383	88	0.283	133	0.488	178	0.169	223	0.417	268	0.24	313	0.161		
44	1.614	89	0.419	134	0.252	179	0.204	224	0.483	269	0.612	314	0.185		
45	0.419	90	0.377	135	0.216	180	0.231	225	0.303	270	0.703	315	0.108		



Fig(2-11) the chart distribution of MD –(Abnormal and Normal)

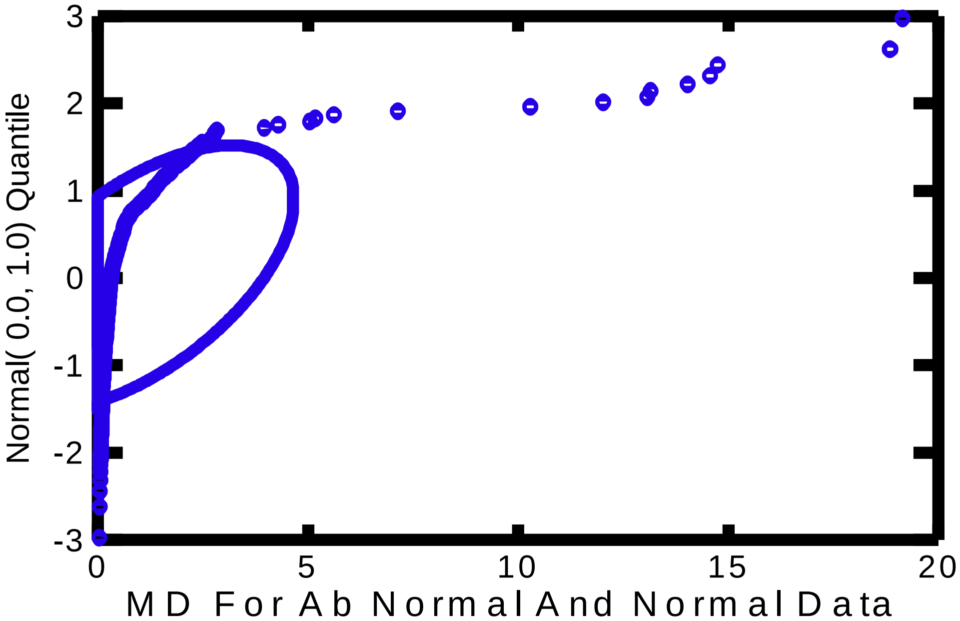


Figure(2-12) the Histogram of all 340 data



Figure(2-13) Convex Hull For MD Of Abnormal And Normal Confidence Ellipse

Fig.(2-11)and(2-13) show the distribution chart of MD value for mixing data(abnormal and normal) group. It is seen that the distributed values of MD between normal and abnormal data are very clear, unstable and unheterogeneous .



`**Figure(2-14) Normal distribution MD Of Mixing data**

Table (2-14) shows the basic statistic of MD value of Mixing data (Normal and Abnormal). It seen that the value of range for mixing data is (19.111) and the standard deviation is (2.42)

Table(2-13)Basic statistic. for Mixing MD value

	Value
N of Cases	**340**
Minimum	**0.038**
Maximum	**19.149**
Range	**19.111**
Arithmetic Mean	**0.997**
Standard Deviation	**2.42**
Variance	**5.856**
p-value	**<0.01**

Discussion :

* MD-(MTS and MTG) are different methods to diagnose multivariate systems.

* MD-(MTS and MTGS) are different methods but the results of the two methods are the same.

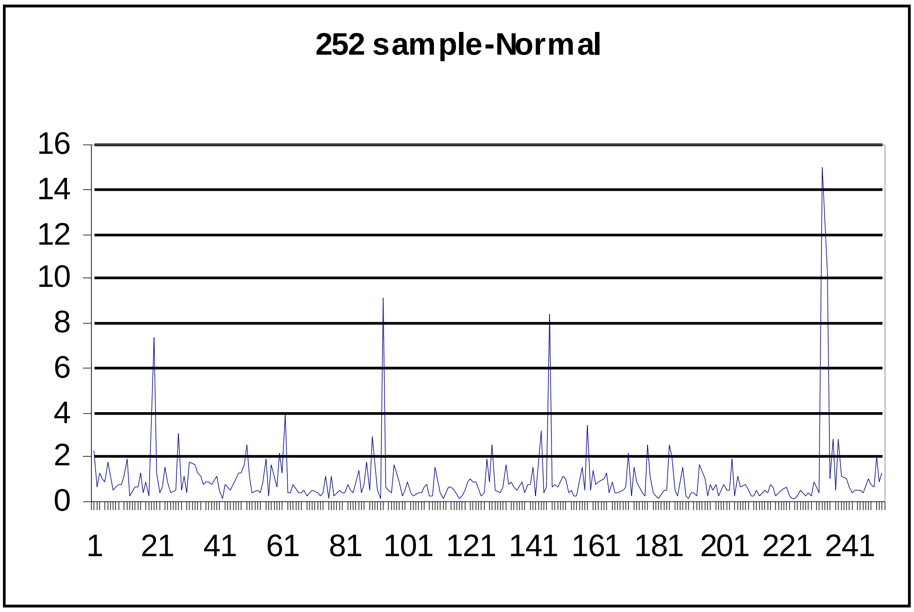
* Defending MD value by MTGS methods is simpler than MD-MTS if partial correlation effects are not significant and there exists a definite order of variables .

* The average value of MD's is always one or close to one.

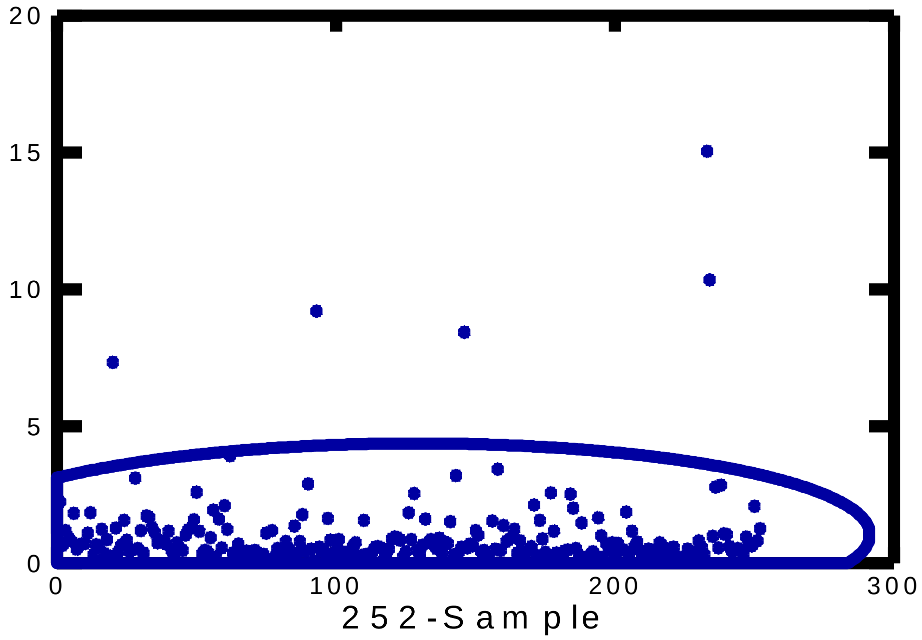
* The means of $(u_{1j}, u_{2j}, ...u_{k_j})$ are zero.

2-5 Removing Abnormal from Normal data

From Table(2-6) and Fig(2-3) and Fig.(2-4) of MD-MTS Normal it is seen that eight points or eight MD from 260 points are out of the normal data , these (8) points can be taken out from all 260 MD data. Then After removing these (8) points the calculation is repeated to compute the MD for (252) samples size , then the value of MD is calculated for (252) sample size as shown in the Table(2-14) and Fig.(2-1) (2-16).



Figure(2-15) Chart of 252 MD Normal



Figure(2-16) **Scatterplote Chart Of Normal MD**

From Fig.(2-15),(2-16), it is shown that there are five points out of the normal points, the value of these points are (7.33,8.44,9.198,10.34,15.04) ,when all MD are sorted it is seen that the 6th point is (3.92458). This value is lees than (7.33) ,it is seen that the difference between point five and point six is too much. Then these five points are removed and calculation is repeated to compute the MD for (247)sample size, the MD value is shown in Table(2-14) and Fig.(2-16),(2-17).

From Fig.(2-17),(2-18) it is seen five points are out of the normal, these values are (4.84, 5.15, 6.27, 8.48, 9.1), therefor the calculation is repeated to compute the MD for (242) sample size, as shown in table(2-14) and Figure(4-19),(4-20).

Discussion :

For each case of sample size there are abnormal points .

1-The first trail sample size 260 it shows that eight points are abnormal , the range is (15.8), and the standard deviation is (1.65)

2- The second trail for (252) sample size it show that five points are abnormal , the range for the second trail is (14.9) and the value of the standard deviation is (1.496)

3- The third trail for (247) sample size shows that although five points are abnormal, the range of this trail is (8.96) and the standard deviation is (1.11).

4- The forth trial for (242) sample size shows that although (5) points are abnormal the rang of this trail is (12.3) and the standard deviation is (1.12) .

5-The fifth trail for (237) sample size shows that although five points are abnormal the range of this trail is (8.808) , and the standard deviation is (1.069).

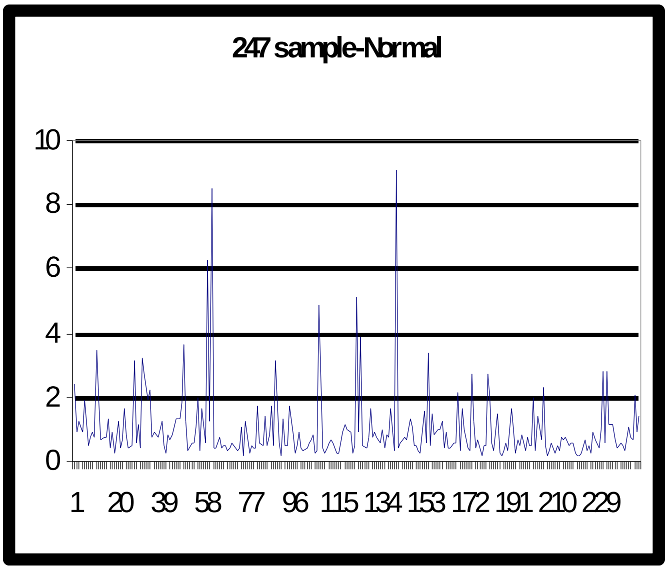
6-The abnormal points for case (252 ,247, 242, 237) are five points ,it mean that it is not necessary to continue to remove the abnormal points.

It is noted that the results of the basic analysis for five trials shows that the smaller value of the standard deviation of MD is for sample size(237), the same result is shown in table(2-15),(2-16). From this trial it is seen that with removing the abnormal point, the value of the standard deviation decreases ,but it is not necessary to continue to remove the abnormal data.

Table(2-14) contains the MDs value of sample size (260, 252, 247, 242,and 237) and the sorting the MDs value for all cases cases.

Table(2-14) MD values For different sample size (260 , 252, 247, 242, 237)

260 MD	252 MD	247 MD	242 MD	237 MD	Sorting 260 MD	Sorting 252 MD	Sorting 247 MD	Sorting 242 MD	Sorting 237 MD
2.1952	2.2538	2.4285	2.4291	2.5553	0.1018	0.1084	0.1340	0.1373	0.1439
0.5887	0.6646	0.8882	0.9883	1.1644	0.1152	0.1268	0.1471	0.1562	0.1658
4.2755	1.2098	1.2781	1.2979	1.4042	0.1187	0.1273	0.1637	0.1755	0.1813
1.1444	0.9757	1.0508	1.1094	1.1565	0.1227	0.1305	0.1689	0.1835	0.1819
0.9437	0.8298	0.9243	0.9549	1.0136	0.1330	0.1408	0.1761	0.1837	0.1865
0.7347	1.8350	1.8970	1.9012	1.8984	0.1463	0.1562	0.1876	0.1841	0.1892
1.7549	0.5110	0.5303	0.5288	0.5602	0.1527	0.1644	0.1900	0.1893	0.1917
0.5069	0.6890	0.7674	0.8201	0.8788	0.1647	0.1841	0.2167	0.2222	0.2347
0.6708	0.7007	0.8783	1.7085	3.3346	0.1690	0.1903	0.2219	0.2265	0.2427
0.6279	0.7292	0.7672	0.8075	0.8475	0.1769	0.1909	0.2327	0.2347	0.2462
0.7138	1.1071	3.4945	4.2236	1.9729	0.1852	0.1931	0.2339	0.2438	0.2586
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
0.3088	0.5475	0.5648	0.5617	0.5775	3.3679	3.1051	3.4168	3.4323	3.1259
0.9495	0.4817	0.5302	0.5415	0.5508	4.2755	3.2076	3.4945	3.5004	3.2452
0.7401	0.3186	0.3221	0.3173	0.3248	4.4323	3.4354	3.6463	3.5637	3.3346
0.6214	0.9748	1.0338	1.0327	1.0366	4.8305	3.9246	3.8833	3.7715	3.8197
1.9360	0.7441	0.7692	0.7667	0.7740	5.7337	7.3388	4.8390	4.0089	4.5659
0.7326	0.6358	0.6612	0.6820	0.7642	6.3613	8.4412	5.1456	4.2236	5.6104
1.1537	2.0947	2.0548	2.0589	2.0660	9.5283	9.1983	6.2701	4.6159	6.6369
9.5283	0.8276	0.8828	0.8866	0.8822	15.4314	10.3421	8.4770	5.6334	7.0150
4.4323	1.2792	1.3853	1.3937	1.4024	15.8802	15.0359	9.0892	12.4159	8.9519



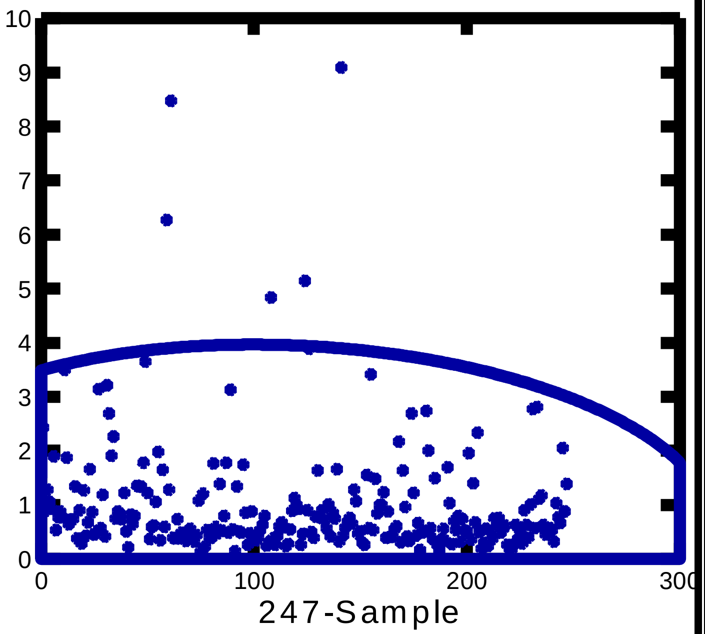
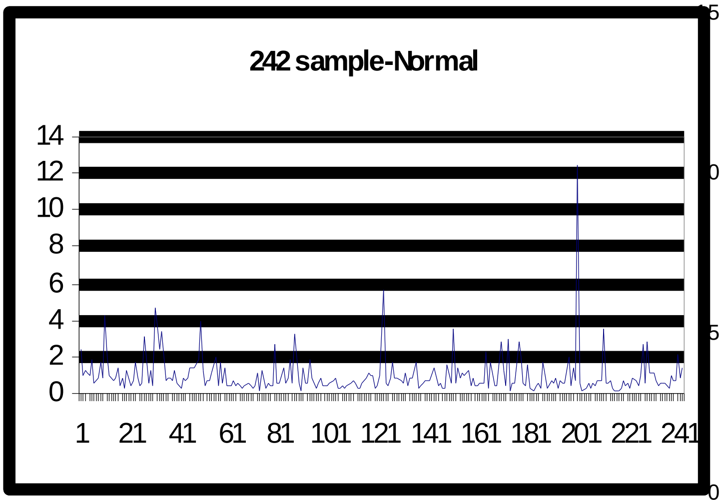


Fig.(2-17) distribution value for (247) MD

Fig.(2-18) distribution value for (247) MD



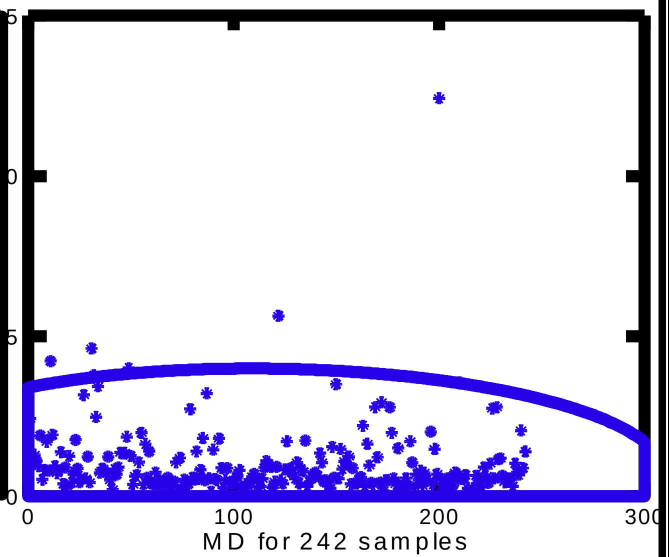
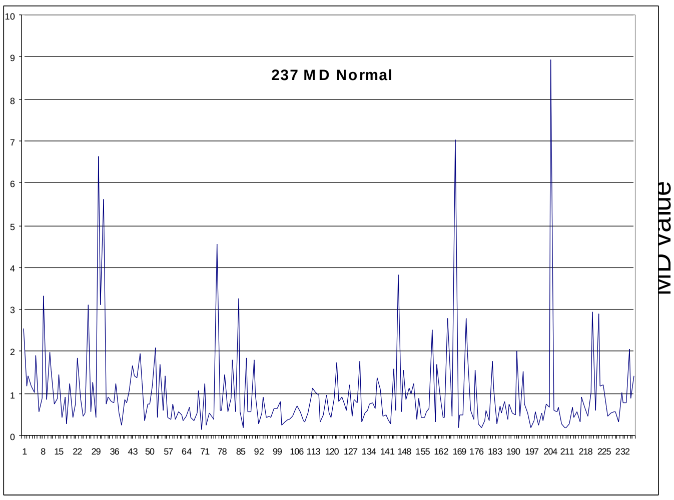


Fig.(2-19) distribution value for (242) MD

Fig.(2-20) distribution value for (242) MD



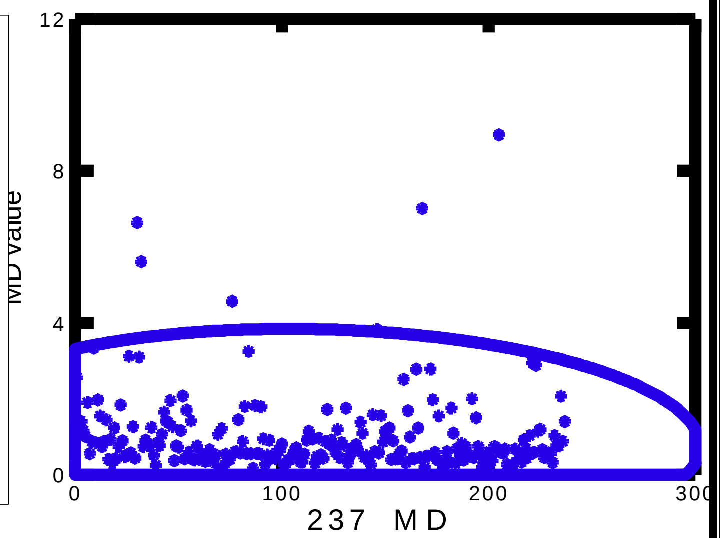


Fig.(2-21) distribution value for (237) MD

Fig.(2-22) distribution value for (237) MD

Tables(2-15) and (2-16) contain the basic statistics of MD values of sample size (260,252,247,242,and 237) It seen that the arithmetic means for all cases are close to one , it seen that case five with sample size (237) have a less standard deviation and range, it mean that removing the abnormal point have a good result on the test.

Table(2-15) shows the Basic statistics of normal data

N of Cases	**260**	**252**	**247**	**242**	**237**
Minimum	**0.1018**	**0.1084**	**0.1340**	**0.1373**	**0.144**
Maximum	**15.8802**	**15.0359**	**9.0892**	**12.4159**	**8.952**
Range	**15.7784**	**14.9275**	**8.9551**	**12.2786**	**8.808**
Sum	**259.0000**	**251.0000**	**246.0000**	**241.0000**	**236**
Median	**0.5488**	**0.6015**	**0.6396**	**0.6500**	**0.66**
Arithmetic Mean	**0.9962**	**0.9960**	**0.9960**	**0.9959**	**0.996**
Standard Error of Arithmetic Mean	**0.1023**	**0.0942**	**0.0707**	**0.0721**	**0.069**
Geometric Mean	**0.6307**	**0.6699**	**0.7216**	**0.7324**	**0.737**
Harmonic Mean	**0.4708**	**0.5184**	**0.5734**	**0.5874**	**0.59**
Standard Deviation	**1.6501**	**1.4955**	**1.1113**	**1.1210**	**1.065**
Variance	**2.7228**	**2.2366**	**1.2349**	**1.2566**	**1.133**
Coefficient of Variation	**1.6565**	**1.5015**	**1.1158**	**1.1256**	**1.069**
Skewness(G1)	**6.3829**	**5.8567**	**4.0181**	**5.3936**	**4.013**
Standard Error of Skewness	**0.1510**	**0.1534**	**0.1549**	**0.1565**	**0.158**
Anderson-Darling Statistic	**41.6800**	**37.4526**	**26.3777**	**25.0978**	**24.822**
Adjusted Anderson-Darling Statistic	**41.8016**	**37.5654**	**26.4588**	**25.1765**	**24.901**
p-value	**<0.01**	**<0.01**	**<0.01**	**<0.01**	**<0.01**

Fig (2-23),(2-24),(2-25) and (2-26) shows that thr normal distribution curve for case sample size (252,247,242,and 237) it shows that the case of sample size 237 have a more normality curve with standard deviation(1.065) and mean (0.9958).

Table(2-16) basic statistics of MD for different sample sizes

#	Sample size	Abnormal point	Standard Deviation	Min	Max	Range
1	260	8	1.6501	0.1018	15.8802	15.7784
2	252	5	1.4955	0.1084	15.0359	14.9275
3	247	5	1.1113	0.134	9.0892	8.9551
4	242	5	1.121	0.1373	12.4159	12.2786
5	237	5	1.065	0.144	8.952	8.808

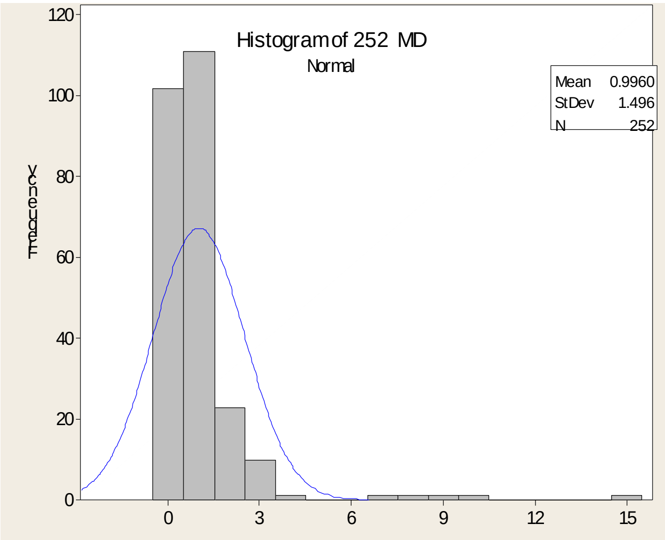


Fig.(2-23) Normal Curve of 252 MD values

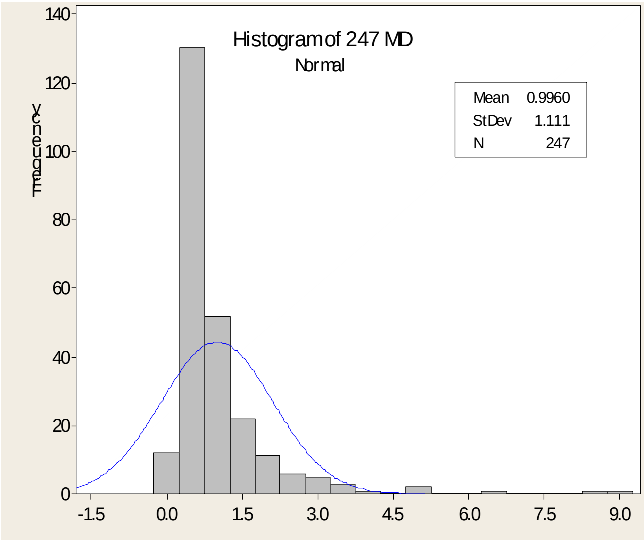


Fig.(2-24) Normal Curve of 247 MD values

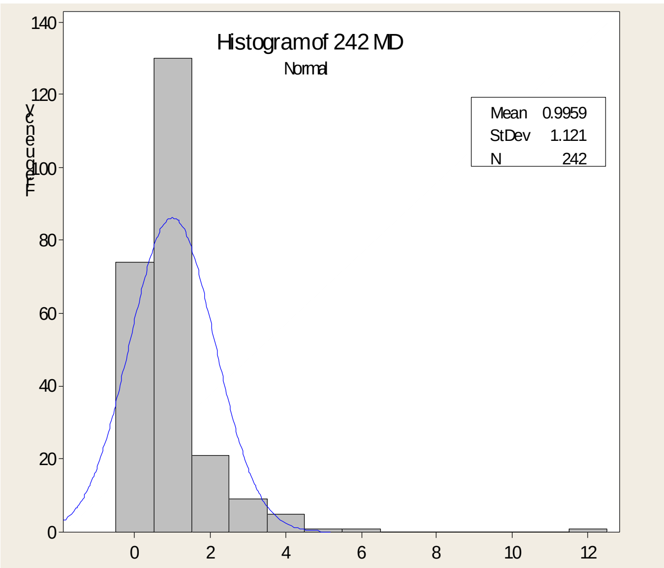


Fig.(2-25) Normal Curve of 252 MD values

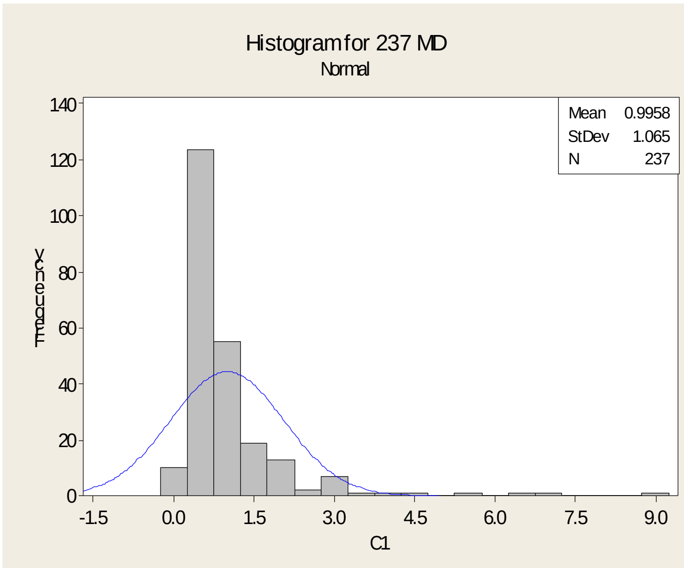


Fig.(2-26) Normal Curve of 237 MD values

Chapter Two
Section Two

Practical Chapter
Orthogonal Array and S/N-Ratio

Chapter Two

Section Two

Minimization of the Number of Variables and Using Orthogonal Array and S/N ratio

This chapter introduces the role of orthogonal array (OA's) and Signal–to-Noise (S/N) ratios in multivariate diagnosis .The data in this chapter contains observation of normal data with sample of size (260) and (12) variables (X1, X2….and X12) as shows in table (2-2) .

Table(2-17) concludes the MD's value for the 16 classes from normal data with sample of size (260) .

Table(2-17) MD for 16 classes of 12 variables from sample size 260 –Normal data

#	MD1	MD2	MD3	MD4	….	MD13	MD14	MD15	MD16
1	1.125	1.122	1.13	…..	……	……	1.171	0.8933	1.061
2	0.871	0.548	1.166	…..	……	……	1	1.0203	1.131
3	1.166	0.969	1.159	…..	……	……	1.13	0.6851	1.093
4	0.675	0.959	1.171	…..	……	……	1.06	0.6246	1.168
5	0.919	1.17	1.092				1.07	0.9269	1.172
6	1.156	1.068	0.633				0.98	1.0286	1.079
7	0.962	0.686	1.162				0.82	0.8362	0.541
8	0.746	0.627	0.84				0.79	0.8592	0.934
9	0.628	1.124	0.8				1.02	0.8951	1.017
10	1.172	1.12	0.917				0.58	1.0058	0.688
11	0.705	0.868	0.536				0.77	1.0748	0.777
12	1.089	1.075	0.622				0.99	1.0272	0.452
13	1.098	1.156	0.666				1.07	0.9806	0.878
14	1.161	0.881	0.93				0.88	0.8017	0.974
15	0.994	1.084	1.096	…..	……	……	0.81	1.1719	0.865
16	0.533	0.541	1.081	…..	……	……	0.87	1.1688	1.172
Average	0.9375	0.937	0.938	…..	…..	…..	0.938	0.94	0.94

Case One :

In this case computing the S/N ratios (The Larger is Better), the mean and the standard deviation , as shown in table(2-18), the output response is the type where it is desired to maximize the result, in the larger –the better characteristic , negative data are not included .

To compute the S/N ratio data in table(2-18) is used and the applied steps have been done by (Minitab-15) program ,the mathematical formula is shown in (2-1). Taguchi Orthogonal Array Design of 16 run with 12 variables is used each with two levels by using $L_{16}(2^{12})$.

The corresponding S/N ratios, mean and standard deviation is shown in Table(2-18), the S/N ratio average response table can be computed as well as the mean with two levels it shown in Table(2-19),(2-20) and Fig.(2-27),(2-28)

$$SNR_i = -10\,Log\frac{1}{n}\sum_{i=1}^{n}\frac{1}{Y_i^2} \qquad \text{………} \quad \mathbf{2\text{-}1}$$

Table(2-18) $L_{16}(2^{12})$MD ,S/N ,Mean and S.D for all 16 runs

No.	A	B	C	D	E	F	G	H	I	J	K	L	MD1	MD2	….	MD16	S / N	Mean	S.D
1	1	1	1	1	1	1	1	1	1	1	1	1	1.13	1.12		1.061	0.44	1.06	0.09
2	1	1	1	1	1	1	1	2	2	2	2	2	0.87	0.55		1.131	-0.8	0.96	0.15
3	1	1	1	2	2	2	2	1	1	1	1	2	1.17	0.97		1.093	-0.8	0.99	0.2
4	1	1	1	2	2	2	2	2	2	2	2	1	0.68	0.96		1.168	-1	0.95	0.2
5	1	2	2	1	1	2	2	1	1	2	2	1	0.92	1.17		1.172	-1.1	0.96	0.18
6	1	2	2	1	1	2	2	2	2	1	1	2	1.16	1.07		1.079	-1.3	0.93	0.2
7	1	2	2	2	2	1	1	1	1	2	2	2	0.96	0.69		0.541	-1.2	0.94	0.19
8	1	2	2	2	2	1	1	2	2	1	1	1	0.75	0.63		0.934	-1.5	0.9	0.18
9	2	1	2	1	2	1	2	1	2	1	2	1	0.63	1.12		1.017	-1.9	0.88	0.21
10	2	1	2	1	2	1	2	2	1	2	1	2	1.17	1.12		0.688	-1.4	0.91	0.18
11	2	1	2	2	1	2	1	1	2	1	2	2	0.7	0.87		0.777	-1.5	0.88	0.15
12	2	1	2	2	1	2	1	2	1	2	1	1	1.09	1.07		0.452	-1.4	0.94	0.19
13	2	2	1	1	2	2	1	1	2	2	1	1	1.1	1.16		0.878	-2.2	0.91	0.24
14	2	2	1	1	2	2	1	2	1	1	2	2	1.16	0.88		0.974	-1.9	0.89	0.21
15	2	2	1	2	1	1	2	1	2	2	1	2	0.99	1.08		0.865	-1.1	0.94	0.19
16	2	2	1	2	1	1	2	2	1	1	2	1	0.53	0.54		1.172	-1.8	0.95	0.24
													Total				-20.6	15	3.007
													Average				-1.28	0.938	0.188

Table(2-19) shows the response of S/N ratios average table of signal to Noise ratios of normal data with two levels and shows the delta , rank and the difference (gain) between two levels and Fig.(2-27),(2-28) shows the S/N ratio chart .

Table(2-19) Response Table for Signal to Noise Ratios

Level	A X1	B X2	C X3	D X4	E X5	F X6	G X7	H X8	J X9	K X10	L X11	M X12
Level 1	-0.899	-1.052	-1.153	-1.270	-1.073	-1.158	-1.268	-1.170	-1.150	-1.288	-1.162	-1.313
Level 2	-1.670	-1.517	-1.417	-1.299	-1.497	-1.411	-1.302	-1.399	-1.419	-1.282	-1.407	-1.257
Delta	0.771	0.465	0.264	0.029	0.424	0.253	0.034	0.229	0.269	0.006	0.245	0.056
Rank	1	2	5	11	3	6	10	8	4	12	7	9
Gain	0.771	0.465	0.264	0.029	0.424	0.253	0.034	0.229	0.269	-0.01	0.245	-0.06

Fig. (2-27) The main effect plot for S/N ratio

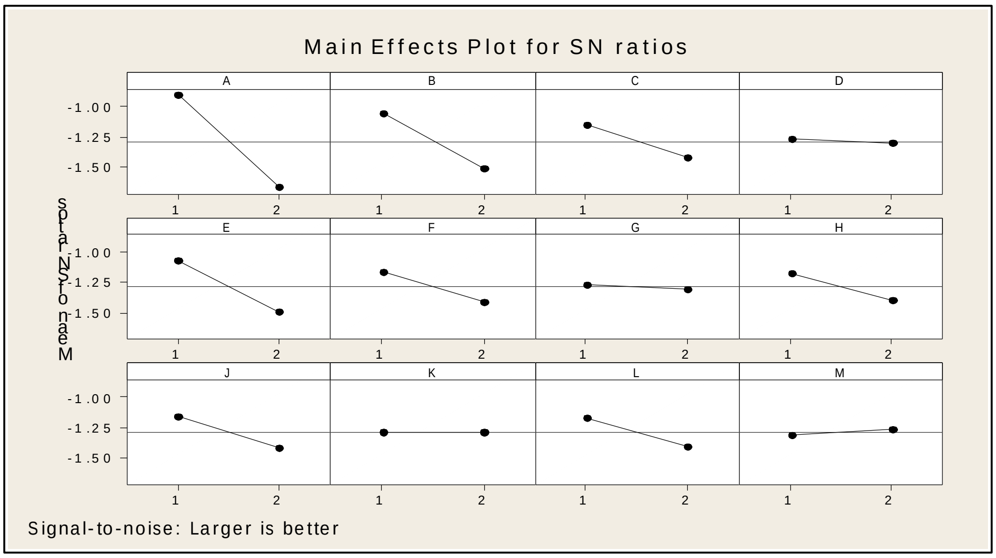


Fig.(2-28) The main effect plot for S/N ratio

From Table(2-19) and Fig.(2-27)and (2-28) it is clear that the variables X_1 , X_2 ,…X_{10} have positive gains. Hence, these variables are considered useful for the future diagnosis.

From this discussion, it is clear that using S/N ratios one can identify the useful set of originate variables in a simple way.

1- The level 1 has a higher SN ratio for (X1, X2, X3, X5, X6, X8, X9, and X11),it means that it has a relatively strong impact on variability .
2- The level 2 has a higher SN ratio for (X10, X12) it has a relatively weak impact on variability .
3- The levels 1 and 2 have roughly the same performance for (X4, X7).

But one can say that the (X4,X7,X10,X12) have the same performance .

i. Variables (X10,X12) are harming discrimination ,they have a negative gain (-0.01,-0.06).

ii. The Variables (X4,X7) do not contribute to discrimination.

iii. The Variables (X1, X2, X3, X5, X6, X8, X9, and X11) contributes greatly to discrimination, these variables are significant and useful, where (X4,X7,X10,and X12) are not significant.

The conclusion is as follows: optimization keep (X1, X2, X3, X5, X6, X8, X9, and X11) and discard all others.

2-6 Computing the optimization:[2][5[15][42]

2-6-1- Optimum of S/N Ratios

The formula (1-42) and (2-2) and response table(2-19) of S/N ratio are used to compute the optimum of S/N ratio as follows :

$$\mathbf{Yopt\ \ S/N = T + (A2-T) + (B2-T) + (C2-T) + (E2-T) + (F2-T) + (H2-T)} + (I2-T) + (L2-T) \quad \text{…. 2-2}$$

Where

Y_{opt} S/N denote the minimum value from Level 1,2 for the (10)variables

T = Total

Then

$$Y_{Actual}\ S/N = T - (A1\text{-}T) + (B1\text{-}T) + (C1\text{-}T) + (D1\text{-}T) + (E1\text{-}T) + (F1\text{-}T) + (J1\text{-}T) + (K2\text{-}T) + (L1\text{-}T) + (M2\text{-}T)$$

Where

Y_{Actual} SN denote the maximum value of original 12 variables in Level 1,and Level 2

Total of SN = -20.6

The Optimize Y_{opt} S/N and Y_{Actual} SN are shown in table(2-20)

And to define the gain by equation (1-43)

By using the Equation (1-44) or (2-4) ,The gain between current and optimal conditional is translated in to (29.13 %) reduction in energy transformation variability, as shown in table (2-20)

$$Variability\ Reduction = 1 - (0.5)^{Gain/6} \qquad \text{.....2-4}$$

Table(2-20) Optimization value of S/N

Average	$Y_{Optim.}$	Y_{Actual}	The Gain	Variability Reduction
-1.28	-2.74	0.2374	2.98	29.13 %

From the above table it is clear that the value of Gain of S/N ratio is (2.98) .

From S/N ratio response table(2-21) it is seen that the level 1 has a higher S/N ratio for all (X1, X2, X3, X5, X8, X9, and X11), it means it has relatively strong impact on variability .

Level (1,2) has roughly the same performance for (X4,X6,X7,X10, and X12).

Then

i.The Variables (X4, X6, X7, X10, and X12) do not contribute to discrimination.

ii. The Variables (X1,X2, X3, X5, X8, X9, and X11) contributes greatly to discrimination, these variables are significant and useful .

Table(2-21) Response Table for Means

Level	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
Level 1	0.962	0.947	0.958	0.94	0.953	0.9429	0.937	0.9468	0.95	0.94	0.95	0.94
Level 2	0.913	0.928	0.917	0.936	0.922	0.9321	0.938	0.9282	0.92	0.94	0.93	0.93
Delta	0.048	0.02	0.041	0.004	0.031	0.0109	0.001	0.0185	0.03	0	0.02	0.01
Rank	1	5	2	11	4	9	12	7	3	10	6	8
Difference	0.0484	0.0198	0.041	0.004	0.0304	0.0108	0.00	0.0186	0.034	0.0	0.019	0.013

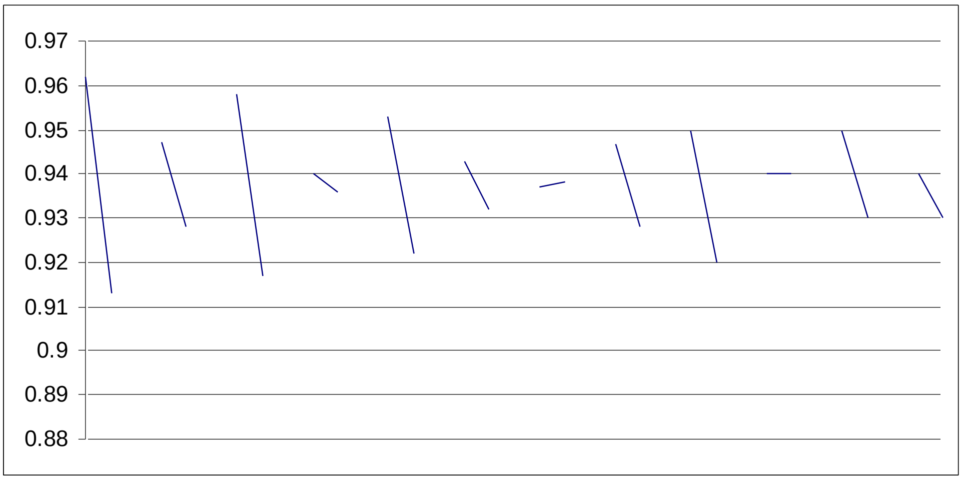


Fig.(2-29) The main effect plot for Mean

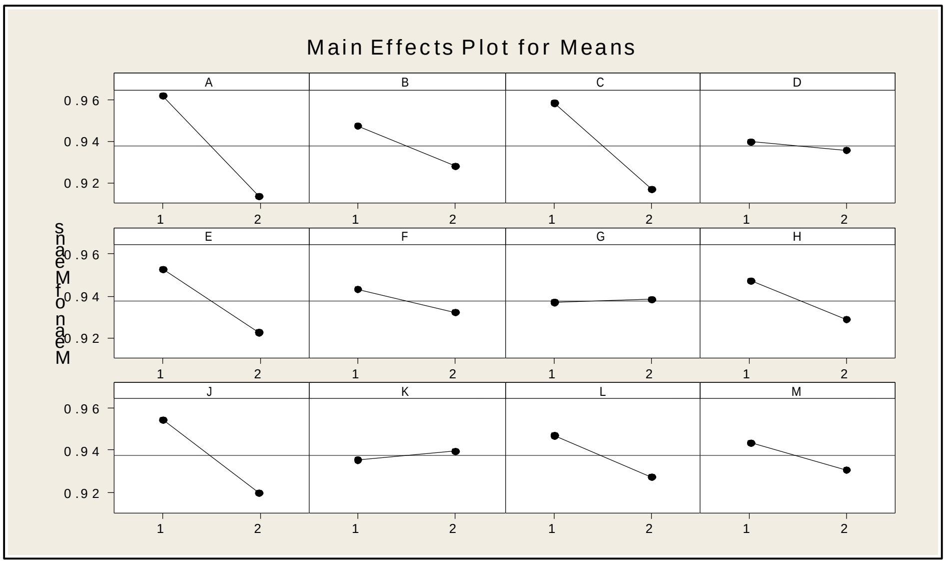


Fig.(2-30) The main effect plot for Mean

2-6-2- Optimum Of mean

The original equation of Y_{opt} .Equation (2-2) is used and the data in Table(2-21) calculate the Y_{opt}, Y_{actual} and Variability Reduction, as shown in table(2-22)

$$\text{Gain} = (\text{Average S/N Ratio})_{\text{Level 1}} - (\text{Average S/N Ratio})_{\text{Level 2}} \quad \ldots 2\text{-}5$$

$$Variability\ Reduction = 1 - (0.5)^{Gain/6}$$

From table(2-22) it is seen that the gain of optimum of mean for(12) variables is (0.2348) and the gain between current and optimal condition is translated into 2.67 % reduction .

Table(2-22) Optimization value of mean

Average	$Y_{Optim.}$ MIN	Y Actual	The Gain	Variability Reduction
0.9375	0.8251	1.0599	0.2348	2.67 %

2-7 Estimating the Model Coefficients

The model coefficients for each S/N and mean are calculated by using MINITAB-15 program .Table(2-24)shows the estimated model Coefficients and Analysis of Variances for SN .

From table(2-19) it is seen that the value of larger delta is (0.77) for variables (X1).But this variable has a smaller P-value , and a larger (T-value) as shown in the table(2-23), but the smaller value of delta is for variables (X10)it has a larger P-value and smaller T-value . From table(2-24) it is seen that the (R- sq) for all 12 variables is equal (91.8 %,). (R-sq(adj)=58.9%),it means that the (R^2) for (12) variables is significant . Table(2-23) shows the comparison the rank or delta of SN response with (P, T –value).

Table(2-23) value Delta, P, T for SN for 12 variables

	Var.	Delta	P-Value	T-Value
Larger	X1	0.77	0.03	3.892
Smaller	X10	0.01	0.977	-0.031

Table(2-24) Estimated coefficients for SN for 12 variables

Term	Coef	T-value	P-value
C	-1.28477	-12.969	0.001
X1	0.38556	3.892	0.03
X2	0.23242	2.346	0.101
X3	0.13189	1.331	0.275
X4	0.01457	0.147	0.892
X5	0.21181	2.138	0.122
X6	0.12641	1.276	0.292
X7	0.0172	0.174	0.873
X8	0.1145	1.156	0.331
X9	0.13453	1.358	0.268
X10	-0.00311	-0.031	0.977
X11	0.12251	1.237	0.304
X12	-0.0278	-0.281	0.797
R –sq =91.8 % S=0.39 R-Sq(adj) = 58.9%			

Table(2-26) shows that the estimated model coefficients of mean of 12 variables . From responsible table(2-21) of mean it is seen that the larger value of delta is (0.0484) for variable (X1) it has a smaller p-value and larger (T-value) as shows in table (2-26), and the smaller delta is for variable (X10) it has a larger P-value and smaller (T-value) value as shows in table (2-26). The mean has a R2 % 97.4 and R-Sq (adj (% 87) it means that R-sq. is significant.

Table(2-25) value Delta, P, T -value for mean

	Var. D	Delta	P-Value	T-Value
Larger delta	X1	0.048	0.01	5.881
Smaller delta	X10	0.001	0.91	-0.123

Table (2-26) Estimated coefficients for mean

Term	Coef	T-Value	P-Value
C	0.938	228.2	0
X1	0.024	5.881	0.01
X2	0.01	2.407	0.095
X3	0.02	4.989	0.015
X4	0.002	0.479	0.665
X5	0.015	3.708	0.034
X6	0.005	1.326	0.277
X7	-5E-04	-0.123	0.91
X8	0.009	2.253	0.11
X9	0.017	4.2	0.025
X10	-0.002	-0.519	0.639
X11	0.01	2.353	0.1
X12	0.006	1.562	0.216
R –Sq =97.4 % S=0.01643 R-Sq(adj) = 87 %			

From tables(2-25) and (2-26), it is clear that :

1-The larger rank has the smallest p-value and vice versa.

2-The largest rank has the largest T- values .and vice versa

Case Two:

The Nondynamic Type systems one of the most important applications of the dynamic SN ratio is to improve the robustness of product of process function within a certain output range.

In nondynamic SN ratio there are two issues .

1- To reduce variability .

2- To adjust the average of the targate.

The dynamic SN ratio are used in the following two situations :

I- If the true levels of severity of all the abnormal data are known.

II- If the true levels of the severity are not known and the working average is used and if the data are not negative.

2-8 Calculating the SN and Sensitivity of MD :

By using the equations (1-33, 1-34, 1-35, 1-36 ,1-37, 1-38) and the data in Table (2-17), the S/N ratio and sensitivity are computed as shows in table(2-27). After that by using the orthogonal way it is possible to calculate response level for S/N ratios and sensitivity table for 12 variables with two levels as shows in table (2-28) and table(2-29), Fig.(2-31) and Fig.(2-33) shows the effects charts for S/N ratio and Sensitivity .

Table(2-27) S/N and sensitivity

S/N	Sensitivity
21.3	0.53551
16.01	-0.32065
13.75	-0.11364
13.68	-0.4436
14.6	-0.38041
13.54	-0.65275
13.7	-0.52757
13.89	-0.96752
12.47	-1.09068
14.02	-0.8558
15.23	-1.09551
13.89	-0.5692
11.69	-0.79025
12.45	-0.98376
13.96	-0.54334
11.77	-0.50215

Table(2-28) Response level for S/N ratio's

Level	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
Level 1	15.1	15	14.33	14.51	15.04	14.64	14.77	14.59	14.44	14.3	14.5	14.16
Level 2	13.2	13.2	13.92	13.73	13.21	13.6	13.47	13.65	13.81	13.94	13.74	14.08
Delta	1.88	1.84	0.41	0.78	1.83	1.04	1.3	0.93	0.63	0.36	0.76	0.08
Rank	1	2	10	7	3	5	4	6	9	11	8	12
Gain	1.88	1.84	0.41	0.78	1.83	1.04	1.3	0.94	0.63	0.36	0.76	0.08

Table(2-29) Response level for Sensitivity

Level	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
Level 1	-0.36	-0.49	-0.40	-0.57	-0.44	-0.53	-0.59	-0.50	-0.42	-0.61	-0.49	-0.53
Level 2	-0.80	-0.67	-0.77	-0.60	-0.72	-0.63	-0.57	-0.66	-0.74	-0.55	-0.67	-0.64
Delat	0.45	0.17	0.37	0.03	0.28	0.09	0.02	0.16	0.31	0.06	0.17	0.11
Rank	1	5	2	11	4	9	12	7	3	10	6	8
Gain	0.45	0.17	0.37	0.03	0.28	0.09	-0.02	0.16	0.31	-0.05	0.17	0.11

From table(2-28) shows that the smaller difference between the two levels are (0.36 ,0.08) for variable (X10 and X12), including the same variables in table(2-19), with smaller value. Therefor the SN Y_{opt} is calculated by using the data in table(2-28), the value of gain of S/N ratio is (11.85) and the variability reduction for S/N ratio is (77.44 %). And the gain for Sensitivity is (2.10) and the variability reduction is (21.54 %) as shows in table (2-30).

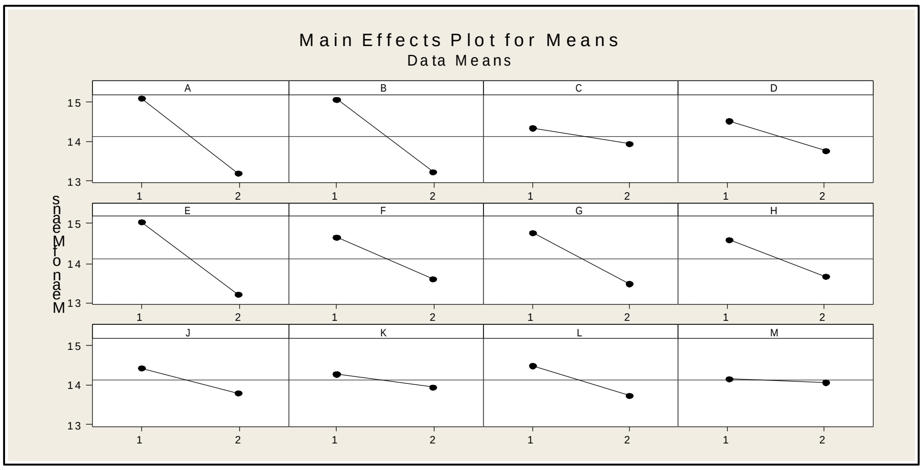


Fig.(2-31) The Main effects S/N Chart

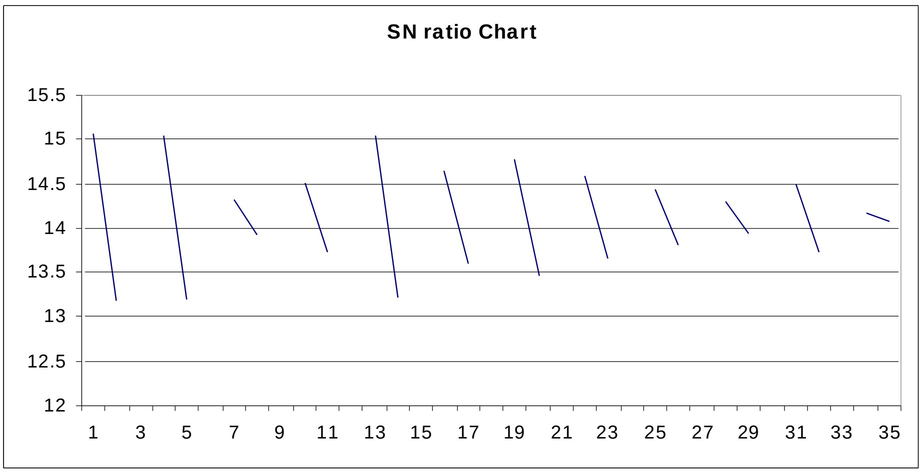


Fig.(2-32) Main effects SN Chart

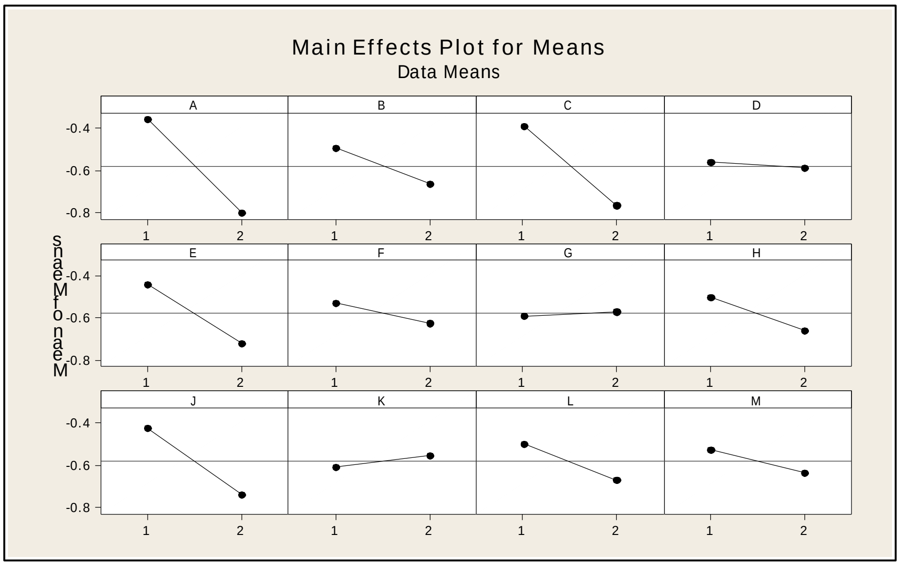


Fig.(2-33) The sensitivity Chart

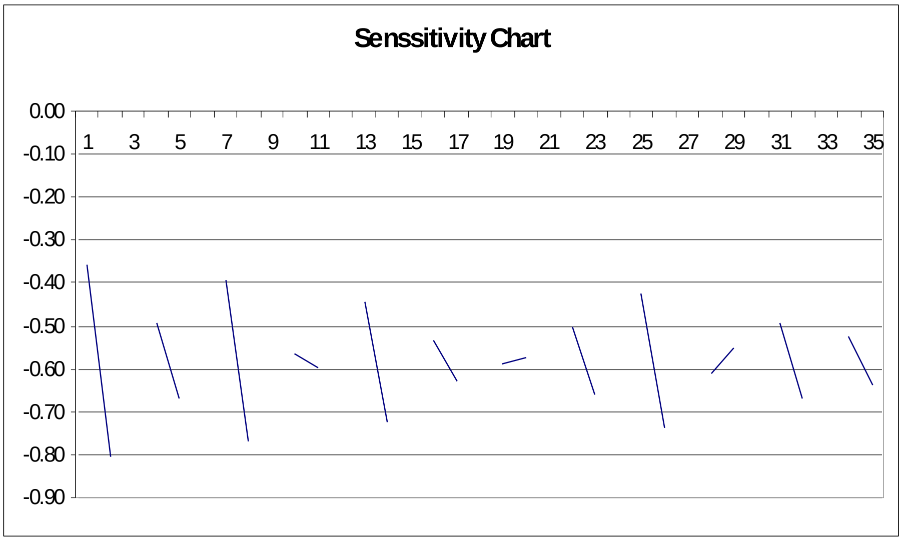


Fig.(2-34) The Sensitivity Chart

Table (2-30) shows that the average, $Y_{opt.}$ Y_{actual} , Gain and Variability Reduction of SN ratio and Sensitivity

Table (2-30) Optimization value of S/N

	SN	Sensitivity
Total	225.94	-9.301
Average	14.12	-0.581
Y_{opt}	8.237	-1.64
$Y_{Actual\ Opt.}$	20.05	0.46
Gain	11.85	2.10
Variability Reduction	74.44 %	21.54 %

2-9 Computing the S/N ratio for the useful variables

Computing the S/N ratio for 10 positive variables :

From S/N ratio average response level Table(2-19) it is seen that the difference value between level 1 and level 2 are positive except two variables (X10 and X12) which are negative .These two variables are not significant and not useful variable therefor to compute the S/N ratio, repeat the same application to calculate the S/N ratio for 10 variables of normal sample size , without variables (X10) and (X12.), MD ‘s value for 16 classes as in table(2-31).

Table(2-31) MD for 16 classes of 10 variables for sample size 260 –Normal data

No.	MD1	MD2	MD3	MD4 …. MD13	MD14	MD15	MD16	SN	Mean
1	1.3	1.03	1.19	….	1.39	0.85	1.21	0.77	1.15
2	0.98	0.41	1.36	….	1.24	1.12	1.28	-1.1	1.02
3	1.39	1.05	1.25	….	1.28	0.77	0.98	-1	1
4	0.66	1.03	1.4	….	1.17	0.43	1.21	-2.5	0.92
5	0.84	1.4	0.9	….	0.81	0.88	1.41	-1.8	0.95
6	1.28	0.96	0.4		1.4	0.31	1.11	-3.4	0.92
7	1.1	0.76	1.38	….	1.02	0.98	0.47	-1.7	0.96
8	0.29	0.73	0.72	….	0.72	0.92	0.87	-3	0.91
9	0.38	1.18	0.67	….	0.74	1.05	1.19	-4	0.83
10	1.4	1.05	0.76	….	0.29	1.16	0.53	-2.9	0.92
11	0.49	0.79	0.45	….	0.82	0.69	0.46	-3.5	0.76
12	1.28	1.16	0.67	….	0.61	1.16	0.18	-5.4	0.9
13	1.29	1.31	0.64	….	1.23	1.05	0.99	-2.3	0.98
14	0.7	0.61	0.98	….	1.02	0.84	0.77	-2.9	0.84
15	1.1	1.04	0.98	….	0.66	1.39	0.93	-2.6	0.97
16	0.53	0.47	1.25	….	0.6	1.4	1.41	-2.2	0.99

Table(2-32) and (2-33) show the response table for S/N ratio and mean for level 1 and level 2 for the 10 variables for normal data .

Table(2-32) Response Table for Signal to Noise Ratios for 10 variables

Level	X1	X2	X3	X4	X5	X6	X7	X8	X9	X11
Level 1	-1.72	-2.46	-1.74	-2.2	-2.41	-2.1	-2.4	-2.02	-2.15	-2.41
Level 2	-3.23	-2.5	-3.21	-2.76	-2.54	-2.85	-2.56	-2.94	-2.81	-2.55
Delta	1.512	0.037	1.472	0.562	0.135	0.753	0.16	0.918	0.661	0.139
Rank	1	10	2	6	9	4	7	3	5	8
Gain	1.512	0.037	1.472	0.562	0.135	0.753	0.16	0.918	0.661	0.139

Table (2-33) Response Table for mean for 10 variables

Level	X1	X2	X3	X4	X5	X6	X7	X8	X9	X11
1	0.978	0.937	0.983	0.949	0.956	0.969	0.94	0.949	0.962	0.924
2	0.898	0.938	0.892	0.926	0.919	0.907	0.935	0.926	0.913	0.951
Delta	0.08	0.002	0.09	0.024	0.038	0.062	0.005	0.022	0.05	0.027
Rank	2	10	1	7	5	3	9	8	4	6
Gain	0.08	-0	0.09	0.024	0.038	0.062	0.005	0.022	0.05	-0.03

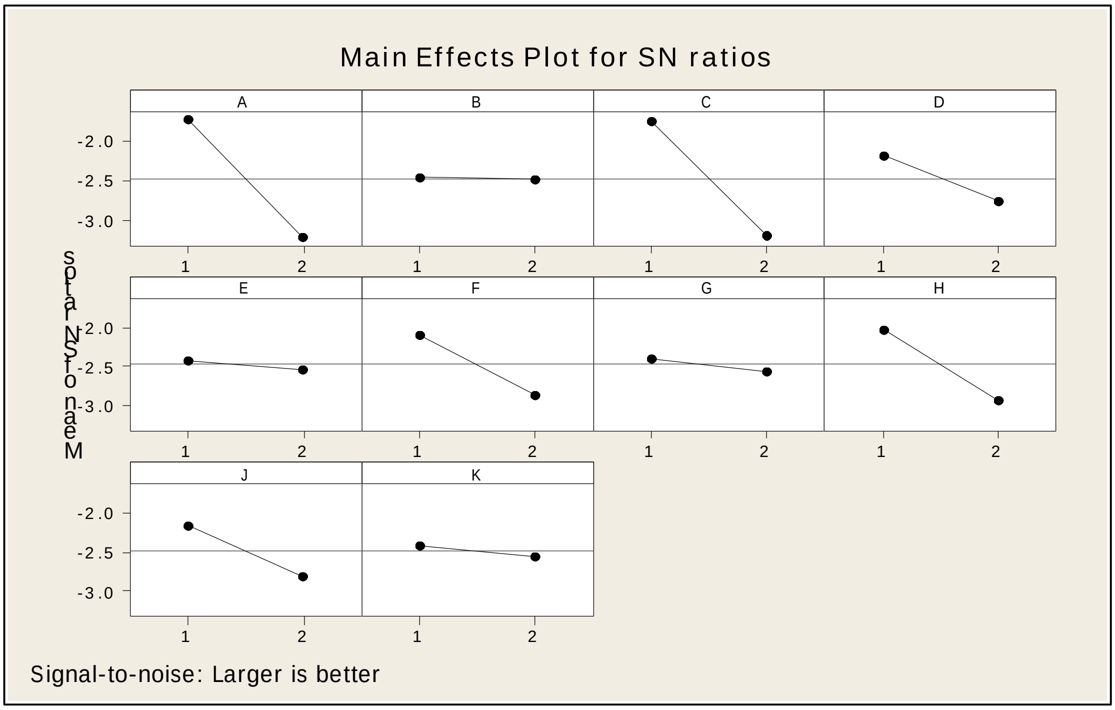


Fig.(2-35) Main effects Chart for SN of 10 factors

From Table(2-32) and Fig.(2-35) it is clear that:

1- Level 1 has a higher SN ratio for all (X1, X3, X4, X6, X8, and X9).

2-Levels 1 and 2 have roughly the same performance for (X2,X5,X7,and X11).

Then the variables (X1, X3, X4, X6, X8, and X9).) contributes greatly to discrimination, these variables are significant and useful. Variable (X2,X5,X7,and X11) do not contribute greatly to discrimination, these variables are not significant and not useful .

Table (2-34) shows that the Estimated Model Coefficient For S/N ratio it clear that the larger gain is for X1 is (1.512) have a larger value of T and smaller value of P and the smaller value of gain is for X2 is (0.037) have a smaller value of T and larger value of P . The R –Sq for 10 variable is (91.3 %) and the R-Sq-(adj) is (73.8 %) that it is mean the analysis for 10 variables is significant .

Table(2-34) Estimated Model Coefficients for SN ratios

Term	Coef	T	P
Constant	-2.5	-14	0
A 1	0.76	4.23	0.01
B 1	0.02	0.1	0.92
C 1	0.74	4.12	0.01
D 1	0.28	1.57	0.18
E 1	0.07	0.38	0.72
F 1	0.38	2.1	0.09
G 1	0.08	0.45	0.67
H 1	0.46	2.57	0.05
J 1	0.33	1.85	0.12
K 1	0.07	0.39	0.71
R –sq =91.3 % S=0.7154 R-Sq(adj) = 73.8 %			

2-10 **Calculating the optimization for useful variables**

After removing the variable (X10, and X12) from all normal sample size (260) then recalculating to determine the SN ratio and mean as shows in table response table of S/N ratio and Mean (2-32), (2-33). After that optimization for SN ratio and mean by using the response Table(2-32) and table (2-33) and calculating the gain and Var. reduction , as shows in compares table(2-35)

Table(2-35) Optimum Table for 12 variables and 10 variables

	Optimum of 12 Variables					**Optimum for 10 Variables.**				
Min	**Average**	**Y Optim.**	**Y Actual**	**The Gain**	**Var. Reduction**	**Average**	**Y Optim.**	**Y Actual**	**The Gain**	**Var. Reduction**
S/N	**-1.28**	**-2.74**	**0.237**	**2.98**	**29.13 %**	**-2.5**	**-5.65**	**0.697**	**6.35**	**51.9 %**
Mean	**0.938**	**0.825**	**1.06**	**0.235**	**2.67 %**	**0.94**	**0.738**	**1.137**	**0.4**	**12.30 %**

Discussion :

1-From table(2-35) ,the gain of optimum S/N ratio's of 12 variables is (2.98), but the gain of 10 variables is (6.35), it means that the gain of S/N ratio's for (10) variables is much better than gain in (12) variables .

2- The gain of optimal mean for 12 variables is equal to (0.235) but for 10 variables is equal to (0.4) .It is means that the gain of optimal mean for 10 variables is much better than the gain for (12) variables.

3- From Estimated Model Coefficients Table(2-24) for S/N ratio's for 12 variables the value of R^2 is equal to (% 91.8) , S= 0.39 and R^2 (adj)= % 58.9 .

But the value of R^2 with 10 variables is (91.3 % , S= 0.7154 and R^2 (adj)= 73.8%) it is seen that the value of (R^2) in both ways is the same , but the R^2 (adj) of the 10 Variables is greater than (R^2 (adj)) of the (12) Variables. It means that removing the two variables from analysis is significant. Table(2-36) shows that the optimum value of (S/N and mean), and (R-Sq, with R-adj) for normal data with 12 variables and 10 variables .

Table(2-36) **compare** table between 12 and 10 variables

	Opt. SN	Opt Mean	R^2	R adj
12 variables	2.98	0.235	% 91.8	% 53.9
10 Variables	6.35	0.4	% 91.3	% 73.8

2-11 Testing the Abnormal data:

Determining the MD values for Abnormal Data from 260 samples.

In this section, it is necessary to analyze and determine the abnormal data from the normal data sample of size (260). From MD table (2-6) and Fig.(2-3) it shows that there are (8) points are abnormal it means that these (8) points are not on the range of normal distance. Those eight points are (4.28, 4.43, 4.84, 5.73, 6.36, 9.53, 15.4, 15.9).

To Calculate the MD for abnormal points using the standard deviation and mean of normal data the MD value as shows in table (2-37).

Table(2-37) The MD Abnormal for eight points

#	1	2	3	4	5	6	7	8	AVERAGE
MD	5.53	59.8	18.3	8.17	9.19	15	43.5	19.7	22.4

From the above table it is seen that the average value of MD is (22.40), this value is greater than one .

For each run of an orthogonal array (OA) , MDs corresponding to the known abnormal conditions or the conditions outside MS are computed .It is not needed to consider the MDs corresponding to the MS (health group) , because it is known that this group is health and the scale constructed is based on the group .Then to calculate the MD for those points the researcher uses the combination orthogonal way for 12 variables and assign variables to a two-levels orthogonal array with the variable combination. For $L_a(b^c)$ Where

a: The number of experiment runs

b: The number of levels o each factors

c : The number of columns in the array

L: Denotes Latin square

Columns of $L_{16}(2^{12})$ Array

Factors: 12 Runs: 16

Then the combination orthogonal MD for abnormal data can be calculated by using the orthogonal array way assign variables to two-level orthogonal array ,define level 1 , to use variable and level 2 not to use variable as shows in table (2-38).

The response MD value is shown in table(2-39) .Therefor the S/N ratio average is calculated for two levels for abnormal are responded as it shown in table(2-40), Table(2-38) **Two level orthogonal array $L_{16}(2^{12})$**

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12												
L16	A	B	C	D	E	F	G	H	J	K	L	M	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
1	1	1	1	1	1	1	1	1	1	1	1	1	Use	Use	Use	Use	Use	Use	Use	Use	Use	Use	Use	Use
2	1	1	1	1	1	1	1	2	2	2	2	2	Use	Use	Use	Use	Use	Use	Use					
3	1	1	1	2	2	2	2	1	1	1	1	2	Use	Use	Use					Use	Use	Use	Use	
4	1	1	1	2	2	2	2	2	2	2	2	1	Use	Use	Use									Use
5	1	2	2	1	1	2	2	1	1	2	2	1	Use			Use	Use			Use	Use			Use
6	1	2	2	1	1	2	2	2	2	1	1	2	Use			Use	Use					Use	Use	
7	1	2	2	2	2	1	1	1	1	2	2	2	Use					Use	Use	Use	Use			
8	1	2	2	2	2	1	1	2	2	1	1	1	Use					Use	Use			Use	Use	Use
9	2	1	2	1	2	1	2	1	2	1	2	1		Use		Use		Use		Use		Use		Use
10	2	1	2	1	2	1	2	2	1	2	1	2		Use		Use		Use			Use		Use	
11	2	1	2	2	1	2	1	1	2	1	2	2		Use			Use		Use	Use		Use		
12	2	1	2	2	1	2	1	2	1	2	1	1		Use			Use		Use		Use		Use	Use
13	2	2	1	1	2	2	1	1	2	2	1	1			Use	Use			Use	Use			Use	Use
14	2	2	1	1	2	2	1	2	1	1	2	2			Use	Use			Use		Use	Use		
15	2	2	1	2	1	1	2	1	2	2	1	2			Use		Use	Use		Use			Use	
16	2	2	1	2	1	1	2	2	1	1	2	1			Use		Use	Use			Use	Use		Use

Table(2-39) Two level orthogonal array $L_{16}(2^{12})$ for abnormal data

	A	B	C	D	E	F	G	H	J	K	L	M	MD1	MD2	MD3	MD4	MD5	MD6	MD7	MD8	SN	MEAN
1	1	1	1	1	1	1	1	1	1	1	1	1	5.5	60	18	8.2	9.2	15	44	20	21	22
2	1	1	1	1	1	1	1	2	2	2	2	2	8.5	0.7	30	12	14	4.3	67	31	5.5	21
3	1	1	1	2	2	2	2	1	1	1	1	2	1	1.4	30	3.2	0.7	1.7	67	31	2.8	17
4	1	1	1	2	2	2	2	2	2	2	2	1	0.6	1	51	1.2	0.6	1.2	111	53	0	28
5	1	2	2	1	1	2	2	1	1	2	2	1	3	3.5	3	252	306	55	10	3.7	13	80
6	1	2	2	1	1	2	2	2	2	1	1	2	2.9	2.8	4.2	302	364	63	10	4.1	13	94
7	1	2	2	2	2	1	1	1	1	2	2	2	147	1.7	3.2	34	1.4	13	55	21	9	34
8	1	2	2	2	2	1	1	2	2	1	1	1	125	1.3	3.3	28	0.7	13	44	17	4.8	29
9	2	1	2	1	2	1	2	1	2	1	2	1	3	7.5	3.3	428	0.4	164	1.7	1.3	-0.3	76
10	2	1	2	1	2	1	2	2	1	2	1	2	3.5	8.9	4.1	513	0.3	194	2.5	1.7	-0.5	91
11	2	1	2	2	1	2	1	1	2	1	2	2	229	4.6	27	1.8	392	9.1	20	25	13	89
12	2	1	2	2	1	2	1	2	1	2	1	1	193	4.6	23	1.5	326	6.2	17	21	12	74
13	2	2	1	1	2	2	1	1	2	2	1	1	158	11	10	242	0.5	91	50	11	3.2	72
14	2	2	1	1	2	2	1	2	1	1	2	2	187	13	12	290	0.6	112	59	14	5.2	86
15	2	2	1	2	1	1	2	1	2	2	1	2	10	2.7	12	32	370	38	13	1.4	11	60
16	2	2	1		1	1	2	2	1	1	2	1	8.6	2.9	9.8	27	309	34	11	1.2	9.5	50

Table(2-40) and Fig.(2-35) one can see that

1- Level 1 has a higher S/N ratios for all (X1, X5, X7, X8, X9, X10, and X11).

2- Level (2) has a higher S/N ratios for all (X2,X3,X4,X6).

3- Levels 1and 2 have roughly the same performance for (X12).

i. Therefor variables (X2,X3,X6) are harming discrimination and have a negative gain

ii. The Variables (X4, X6,X12) do not contribute to discrimination.

iii. The Variables (X1, X5, X7, X8, X9, X10, and X11) contribute greatly to discrimination, these variables are significant and useful in the analyses and (X4, X6,X12) are not significant or not useful

Table(2-40) Response table for Signal to Noise Ratios For AbNormal

Level	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
1	8.6	6.6	7.2	7.5	12.2	7.384	9.14	9	8.9	8.6	8.3	7.8
2	6.6	8.6	8	7.7	3.04	7.835	6.08	6.2	6.3	6.6	6.9	7.4
Delta	2.1	2	0.9	0.2	9.13	0.451	3.07	2.9	2.6	2.1	1.4	0.4
Rank	6	7	9	12	1	10	2	3	4	5	8	11
Gain	2.1	-2	-0.9	-0.2	9.13	-0.451	3.07	2.9	2.6	2.1	1.4	0.4

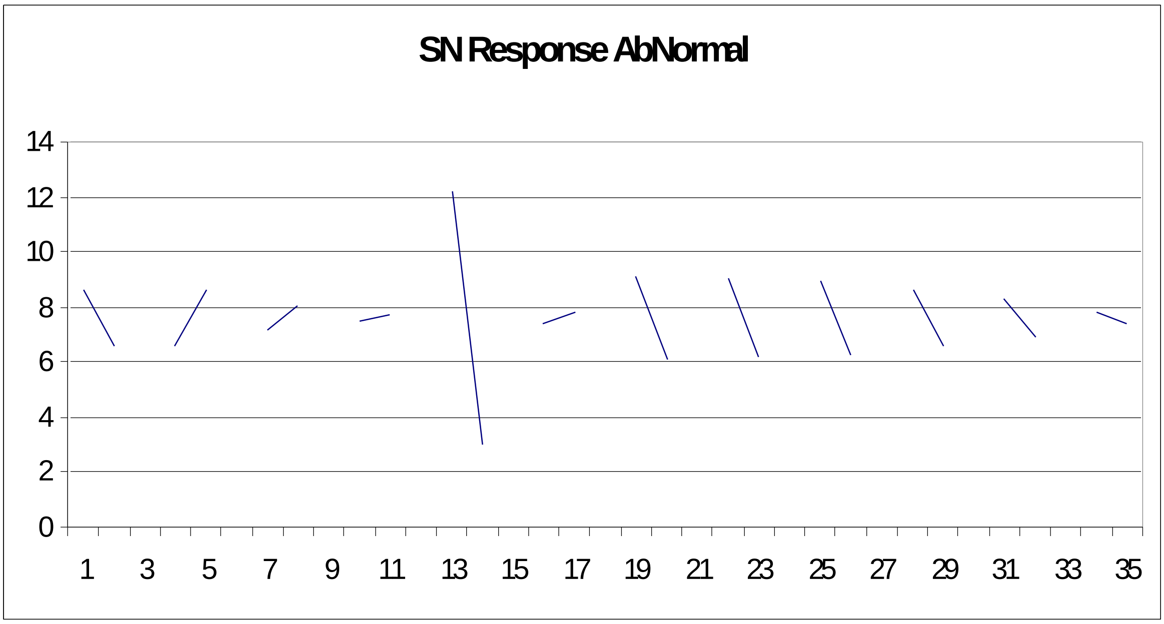


Fig.(2-36) S/N ratio response Chart for Abnormal sample

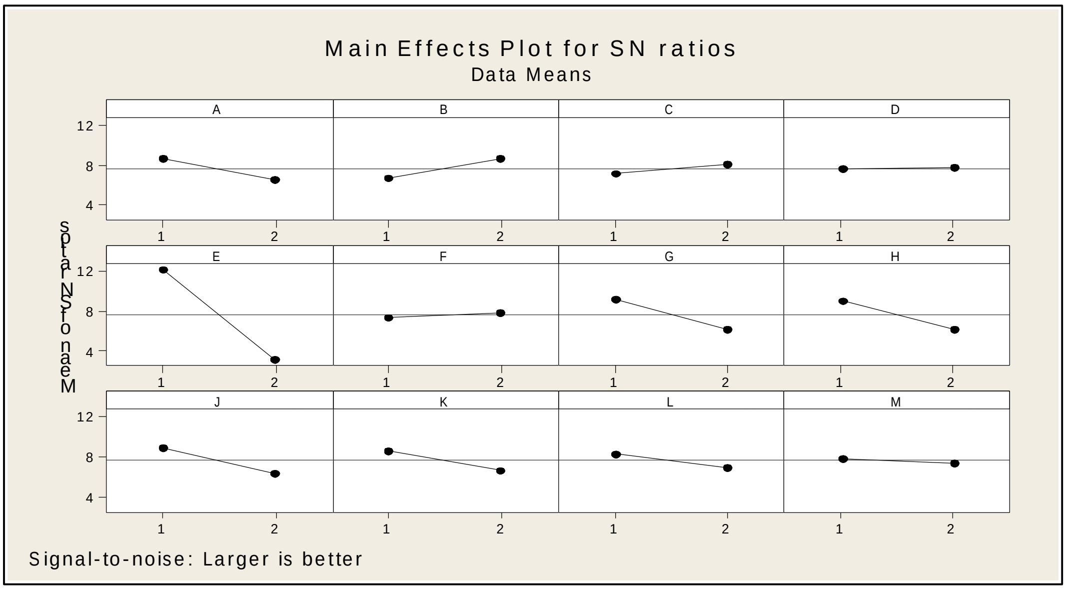


Fig.(2-37) S/N ratio response Chart for Abnormal sample

Table (2-41) and fig(2-37) shows that the response of mean for abnormal data

Table(2-41) Response Table of Mean of Abnormal

Level	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
1	41	52	44	68	61.3	48.05	53.4	56	57	58	57	54
2	75	63	71	48	54.1	67.33	62	59	59	57	58	62
Delta	34	11	26	20	7.15	19.28	8.6	2.9	1.7	0.5	0.5	7.7
Rank	1	5	2	3	8	4	6	9	10	12	11	7
Gain	-34	-11	-26	20	7.15	-19.28	-8.6	-2.9	-1.7	0.5	-0.5	-7.7

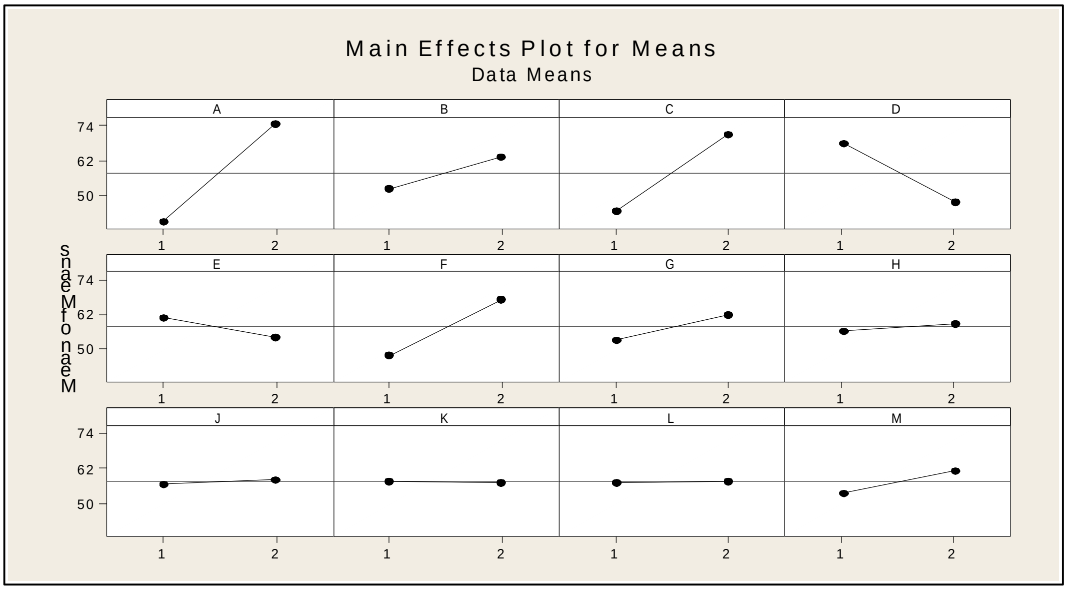


Fig.(2-38) Response AbNormal Mean Chart

2-12 Mixing the Abnormal with Normal data

Can calculating S/N ratio can be calculated by mixing or combining these eight abnormal sample with eight normal samples from 260 samples, then recalculate the MD as shows in table(2-42) concludes the value of MD- MTGS of 16 samples; eight samples are abnormal and eight samples are normal .

From the table it seen that the average of all MTGS is equal to (0.938), but the average of the first eight abnormal samples are equal (1.147), and the second eight samples for normal are equal (0.728). It seen that there is a different between two averages the average of abnormal is grater than one and the others average of MD is equal to unity, or one as shown in table(2-43), then the SN ratio is calculated and level 1,2 is responded as shown in table(2-30).

Table(2-42) MD's value of 16 samples (Normal and Abnormal)

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
MD	1.16	1.17	1.15	1.17	1.17	1.17	1.12	1.06	0.76	0.62	0.42	0.96	0.44	0.71	0.91	1.01

Table (2-43) the average value of MD(ab normal and normal)

	Average
Average of Abnormal sample	1.147
Average of Normal sample	0.728
All average	0.938

By orthogonal methods with ($L_{16}2^{12}$) Array we calculate the value of S/N ratios and mean response ,as shows in table(2-45),(2-46), from table (2-44) it seen that hat four S/N ratios are positive, these values are (2.95 , 2.44 , 4.22 , 2.09), and the rest are negative.

Table(2-44) Two level orthogonal array $L_{16}(2^{12})$ ABNORMAL AND NORMAL

A	B	C	D	E	F	G	H	J	K	L	M	MD1	MD2	MD3	…….	MD15	MD16	SN	Mean
1	1	1	1	1	1	1	1	1	1	1	1	1.16	1.92	1.02		1.26	1.08	2.95	1.7
1	1	1	1	1	1	1	2	2	2	2	2	1.17	1.12	2.01		0.83	1.21	-11	1.33
1	1	1	2	2	2	2	1	1	1	1	2	1.15	1.5	1.65		1.42	1.68	2.24	1.43
1	1	1	2	2	2	2	2	2	2	2	1	1.17	1.47	1.81		2.29	1.97	4.42	1.84
1	2	2	1	1	2	2	1	1	2	2	1	1.17	1.98	0.34		2.61	2.2	-4.8	1.38
1	2	2	1	1	2	2	2	2	1	1	2	1.17	0.57	1.83		1.3	2.34	2.09	1.62
1	2	2	2	2	1	1	1	1	2	2	2	1.12	1.81	1.7		2.1	1.79	-1.9	1.63
1	2	2	2	2	1	1	2	2	1	1	1	1.06	1.53	1.32		0.26	0.35	-4.7	1.04
2	1	2	1	2	1	2	1	2	1	2	1	0.76	0.42	0.22		0.13	0.2	-16	0.25
2	1	2	1	2	1	2	2	1	2	1	2	0.62	0.7	0.31		0.14	0.22	-11	0.37
2	1	2	2	1	2	1	1	2	1	2	2	0.42	0.32	0.27		0.15	0.44	-13	0.34
2	1	2	2	1	2	1	2	1	2	1	1	0.96	0.44	0.79		0.64	0.19	-12	0.43
2	2	1	1	2	2	1	1	2	2	1	1	0.44	0.3	0.16		0.06	0.12	-20	0.18
2	2	1	1	2	2	1	2	1	1	2	2	0.71	0.54	0.85		0.53	0.54	-7.4	0.59
2	2	1	2	1	1	2	1	2	2	1	2	0.9	0.19	0.37		0.43	0.12	-11	0.41
2	2	1	2	1	1	2	2	1	1	2	1	1.01	0.21	0.35		0.84	0.55	-11	0.46

By the same way calculate the S/N ratios and mean table for mixing data (normal and abnormal) with two levels for 12 variables are calculated as shows in table (2-45), Fig.(2-38) and Fig.(2-39)

Table(2-45) Response Table for Signal to Noise Ratios-Abnormal and Normal

Level	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
1	-1.3	-6.8	-6.3	-8.2	-7.2	-7.9	-8.4	-7.7	-5.4	-5.6	-6.5	-7.6
2	-13	-7.3	-7.8	-5.9	-6.8	-6.1	-5.6	-6.3	-8.7	-8.4	-7.5	-6.4
Delta	11.4	0.5	1.5	2.32	0.38	1.83	2.78	1.39	3.29	2.88	1.07	1.27
Rank	1	11	7	5	12	6	4	8	2	3	10	9
Difference	11.4	0.5	1.5	-2.3	-0.4	-1.8	-2.8	-1.4	3.29	2.88	1.07	-1.3

Table(2-46) Response Table for Mean of Normal and Abnormal

Level	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
1	1.5	0.96	0.99	0.93	0.96	0.9	0.9	0.91	1	0.93	0.9	0.91
2	0.38	0.91	0.88	0.95	0.92	0.97	0.97	0.96	0.88	0.95	0.98	0.96
Delta	1.12	0.05	0.11	0.02	0.04	0.07	0.07	0.05	0.12	0.02	0.08	0.05
Rank	1	8	3	11	10	5	6	9	2	12	4	7
Diff.	1.12	0.05	0.11	-0	0.04	-0.1	-0.1	-0	0.12	-0	-0.1	-0.1

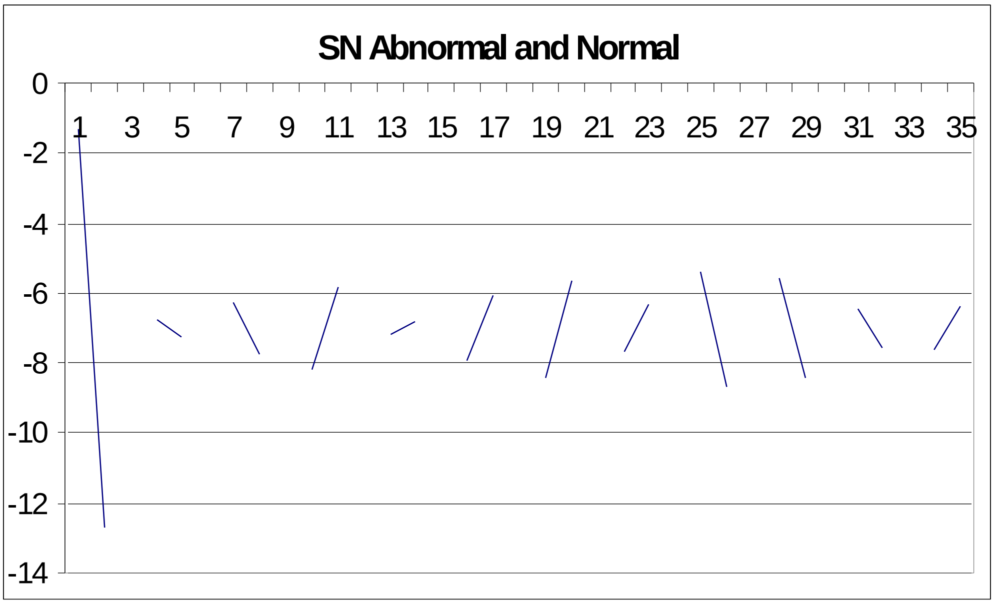


Fig.(2-39) Main effect S/N for Abnormal and Normal

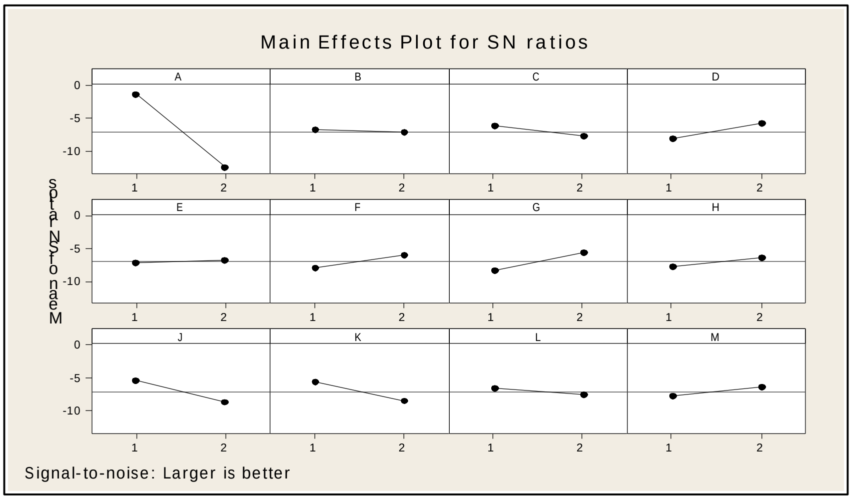


Fig.(2-40) Main effect S/N for Abnormal and Normal

Table(2-45) and Fig.(2-37) show that :

1- S/N rations in Level 1 has a higher SN ratio for all (X1, X2, X3, X9, X10, and X11)

2- S/N ratios in Level (2) has a higher SN ratio for All (X4, X5, X6, X7, X8,and X12) .

3- But one can say that (X2, X5) have the same roughly same performance for (X2, X5).

i. Therefor variables (X4, X5, X6, X7, X8, and X12) do not contribute to discrimination which is a are harming discrimination ,it has have a negative gain and not useful.

ii. Variables (X1, X2, X3, X9, X10, and X11) contribute greatly to discrimination, these variables are significant and useful .

From Table(2-45) and Fig.(2-39) it is seen that six variables from 12 variables have positive gains and hence they are considered useful variables ,these variables are ((X1, X2, X3, X9, X10,and X11).

Discussion:

1- From the above S/N ratios analysis and response level it is seen that six variables from 12 variables have a positive gain these variables are (X1, X2, X3, X9, X10,and X11), these variables are very important and useful in analysis .

2- About variables (X5, X8) they are useful but not very important variables.

3- Therefor Variables (X4, X6, X7,and X12) do not contribute to these variables and discrimination is harming, these variables are not useful and not significant in the analyses.

Chapter Three

Chapter Three
Conclusions and Recommendations

3-1 CONCLUSIONS AND SCOPE FOR FUTURE WORK

In this chapter the conclusions and the basic mission which we have found through this study can be summed up.

1- The MD-MTS and MD-MTGS are two different methods that use Taguchi Methods and Mahalanobis distance to diagnose multivariate systems. The two methods have the same results given MTGS method is simpler than MTS method, with the knowledge that the two approaches have the same complications and have the same results .

2- The average of MD-(MTS and MTGS) value is equal to one or close to one but the average value of MD-Abnormal is greater than one, and

3- The average of vectors $(u_{1j}, u_{2j}, u_{3j}, \ldots u_{kj})$ are equal to zero and the average of $\frac{u_{1j}^2}{s_1^2}, \frac{u_{2j}^2}{s_2^2}, \ldots \frac{u_{kj}^2}{s_2^2}$ are equal to one.

4-The value of optimal gain of (S/N) ratio for (10) variables is (6.35) with but for (12) variables is (2.98) it seen that the gain with 10 variable are larger than if there are (12) variables, it means that we discard unuseful variables.

5- The value of optimal gain of (mea) for (10) variables is (0.4) with but for (12) variables is (0.235) it seen that the gain with 10 variable are larger than if there are (12) variables,

6- The value of R-Sqr for normal data for (12 and 10) variables are equal, but (R^2 (adj)) for (10) variables are larger than if there are with (12) variables, it means that the variable X10 and X12 are not useful in analysis . . .

5-The larger rank has the smallest p-value, and have the larger(F and T) value vice versa.

6- For the normal data the variables (X1, X2, X3, X5, X6, X8, X9,and X11) are useful variables to analyze, and can be used to diagnostic

7- For the abnormal data the Variables (X1, X5, X7, X8, X9, X10 and X11) are very important and useful variable to analysis, therefor these six variables (X1,X5,X7,X8,X9, and X11) are very useful to the result of the analysis .

8- Variable (X10) is useful in abnormal, and (X12) is not useful in the (Normal and Abnormal) it means that the variable (X12) is not useful..

8-2 Recommendations

The main recommendations are concluded in these points:

1- For analyzing the component of water we propose analyzing (10) variables in place of (12) variables it means that variable (X10 and X12) are not useful in analysis.

2- Analysis of Variable (X1, X5, X7, X8, X9, and X11) are very important to decide if the sample is normal or abnormal it must be depending on the result of these variables..

3-Each variable has a special chemical analysis, and for each analysis there are a especial cost. It means that decrease the number of variables to test , decrease the total cost of analysis .

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أن تصميم Á نظام معلومات جيد د متعددة ، مطلب مهم لنجاح و تنفيذ اي نشاط لصانع القرار.

كثير من حالات لا يمكن تحقيق الهدف في البحوث و الدراسات العلمية و بالرغم من توفير معلومات مهمة و جوهرية لانه استعمالها غيرصحيحة بالمعنى الصحيح ،

في حياتنا اليومية نحتاج الى منتجات بنوعية عالية ، لذا فمن الضروري ان تكون هناك انظمة معلومات عن النوعية لصنع قرارات و تنبؤات رصينة على النوعية .

ويمكن لنظام متعدد الابعاد أن يكون نظام تفتيش او يكون نظام طبي (**Medical diagnosis**) و أنه المهم أن يكون هناك معيارللقياسات. و يمكن من خلاله قياس درجة لحالة غير الطبيعية (**Abnormal**) فمثلا في نظام التفتيش ان معيار قياس غير الطبيعي يشير الى حدود قبول المنتج و في المجال التشخيص الطبي ان درجة غير الطبيعي تمثل في شدة المرض.

هذه الاطروحة تقدم اساليب لتطوير و تحسين معيار قياسات متعددة الابعاد من خلال دمج المفاهيم الرياضية و الاحصائية مثل المسافة مهالانوبيس **(Mahalanobois Distance-MD**

و طريقة **(Mahalanobois -Taguchi-System - MTS)**

و طريقة **(Mahalanobois -Taguchi-Gram-Schmidt- MTGS)**

تتضمن هذه الاطروحة دراسة (تحليل و فحص المركبات الماء) ،من خلال هذه دراسة تمكن الباحث تحليل المتغيرات باستخدام طريقة (Taguchi).

تعتبر طريقة تاكوشي اداة لحل مشاكل ان طريقة تعتمد على تصميم و تحليل التجارب الاحصائية تم تطبيقه بشكل خاص، في تصميم المعلمات العملية الانتاجية للحصول على الحالة المثلى لمنتجات هذه العملية .ثم تنفيذ التجربة بأستخدام الخطة التجريبية القياسية و التي لها ($L_{16}2^{16}$) و بشكل مجموعات مرتبة أو مصفوفات متعامدة لكي نحصل على معلمات عملية بـ شكل الامثل من خلال استخدام تحليل نسبة الاشارة Á (S/N to ratios noise) .

و يمكن تقسيم الهدف الرئيس للاطروحة الى مجموعة من الاهداف الفرعية:

1- ان طريقة (Taguchi Method) طريقة جديدة و التي عبارة عن اسلوب حصين والتي **(Mahalanobios distances-MD)** تخدم لتقيم معيار قياس الحالة غير طبيعية .

2-استخدام المصفوفات المتعامدة **(Orthogonal Array- OA)** نسبة اشارة الى الضبابية (S/N) لتحديد المتغيرات المهمة .

3- عرض و تقديم أنظمة متعددة الابعاد ، مثل نظام تفتيش (Inspection) أو نظام التشخيص الطبي (Medical diagnosis) و التي تعتبر ذات الاهمية حيث يمكن من خلاله قياس الحالة غير الطبيعية .

4-الطرق المقترحة يمكن تطبيقها في مجالات عديدة منها صناعية ، هندسية، انتاجية .....

پوخته

وەك ئاشكرایە پاپەند بوون بە چەندین بنەمای جۆراو جۆر لە دارشتنی پلانی تۆكمەدا بۆ سیستەمیكی زانیاری كارێكی باشە بۆ هەموو چالاكیەك ، بەمەبەستی گەیشتن بە بریاری گونجاوی تەواو. هەرچەندە زۆرجار لە دارشتنی سستەمی زانیاریدا نەمان توانیوە بە ئەنجامی تەواوبگەین لەگەلْ ئەوەشدا زانیاری زۆر باشمان لە بەر دەستابووە .وەك دەزانین ئەو زانیاریانە لە پێكهاتەی چەندین گۆرانەوە دروست دەبێت وە پی دەوترێت سستەمی چەندین دوری یان چەندە لایەنە.

روژ لە دوای رۆژ داخوازیەكان واپێویست دەكات كە چۆنیەتی كالاًو پێداویستە بەرهەمهێنەرەكان باش و بەرز بێت بۆ ئەم مەبەستە پێویست دەكات سستەمێكی زانیاری زانستیمان لە بەر دەستابێت تاوەكو بتوانین بریاری زانستی بدەین و وە پێشبینی بكەین .

دەتوانین لەهەموو بوارەكاندا سستەمی چەندین دوری چەندە لایەنە بەكاربهێنرێت وەك رژێمی چاودێری كردن وبە دواداچون یان سستەمی دەست نیشانكردنی تەندروست و نادروست، گرنگ لە وەدا ئەوەیە كە پێوەرەكان دەستنیشان بكرێت و بە تایبەتی پێوەرە نادروستەكان، لە بواری سستەمی چاودێری و بەدوادا چوندا، پێوەرە نادروستەكان هیمایە بۆ سنوری رەزامەندی بەرهەم هێنەرن بەلاًم لە بواری تەندروستیدا پێوەری نادروست بەمانای پیوەرە لە گرژی نەخۆشیدا.

ئەم تێزی دكتورایە هەندی پێوەری چەندین دووریمان پێشكەش دەكات بە بەكارهێنانی رێگەی بیركاری و ئامارەوە وەك پێوەری (دووری) (مەهالانۆبیس-Mahalanobis) سستەمی (Mahalanobis-Taguchi System-MTS) وریگەی (Mahalanobis-Taguchi–Gram Schmidt –MTGS

ئەم تێزە بریتیە لە دوو جۆر لێكۆلینەوە، یەكەم بریتیە لە شیكاركردنی پیكهاتەی ئاو وە دووەم لێكۆلینەوە بریتیە لە گرفتی دەستنیشان كردنی تەندروستی، بە هۆی بە بەكارهێنانی رێگای

(Taguchi – تاگۆشی) وە دەتوانین بەرنامەیەکی باش دابرێژین بۆ چارەسەری ئەم کێشانە و باشترین سستەمی پلان بۆ رژێمی پارامیتەرەکان دابنێین.

ئەم تاقیکردنەوەیە بە بەکارهێنانی تاقیکردنەوەی ڕێگەی تاگۆشی و بە دەلالەتی $L_{16}(2^{16})$ بۆ چارەسەر کردنی پارامیتەرەکان لە ڕێگەی ڕێژەی تێکرای هێماکان (S/N) بە مەبەستی دەست نیشانکردنی پارامیتەرە زاڵ بووەکان و کەم کردنەوەی جیاوازی ئەو پارامیتەرانەی کە دەکەونە دەرەوەی کونترۆڵی (S/N) .

دەتوانین لە چەند خاڵێکدا بیرۆکەی تاگۆشی دابرێژینەوە .

1– تاگۆشی بەهۆی سود وەرگرتن لە ڕێگەی (دوری- Mahalanobis) توانی نزیکراوەیەکی بەهێزتر پێشکەش بکات.

2– بەکارهێنانی (Orthogonal Array -OA) و تێکرای (S/N - Ratio) ئاماژەیە بۆ نارِوونی و دەست نیشانکردنی سفەتە باشەکان لە لێکۆلینەوەکاندا.

3– خستنە رووی سستەمی چەندین دووری وەك سستەمی چاودێری یان دەست نیشانکردنی تەندروستی ،کە ئەمە خۆی لەخۆیدا گرنگی هەیە و بە هۆیەوە دەتوانرێت نا تەندروستەکان پێوانەی بۆ بکرێت.

4– دەتوانرێت ڕێگەی پێشنیاز کراو لە چەندین بواردا بەکاربهێنریت وە بواری پیشەزازی کشتوکاڵی و تەندروستی ..... .